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ANALYTICAL POSITIONING OF AIDS TO NAVIGATION.(U)
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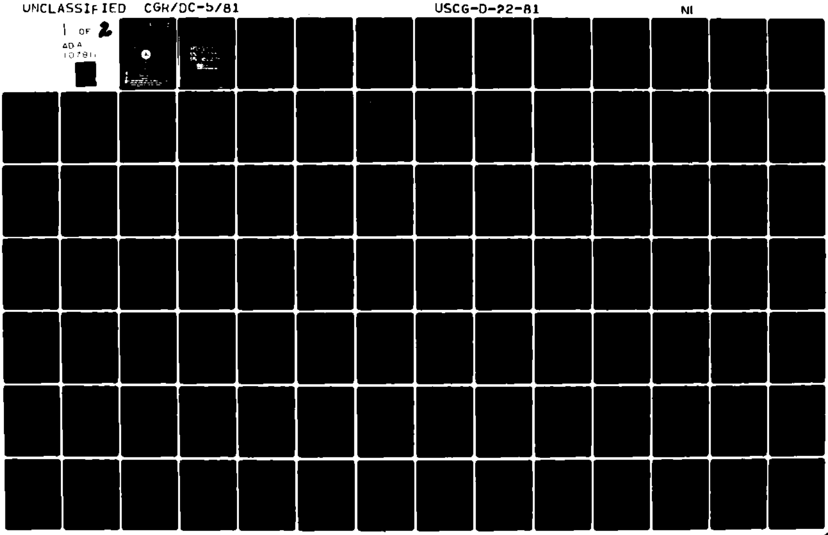

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16. Abstract

Sophisticated systems such as the Global Positioning System (GPS) and the Inertial Positioning System (IPS) are not yet economically justified for widespread Coast Guard use in positioning aids to navigation. To improve accuracy and precision in positioning until such systems are justified, alternative procedures have been explored. Accuracy and precision are addressed to create various mathematical, statistical and operational procedures immediately applicable to the Coast Guard. This adventure into marine geodesy can help the Coast Guard employ error minimizing positioning procedures until more precise methods and reliable techniques are available.

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METRIC CONVERSION FACTORS

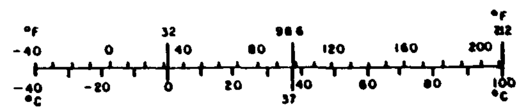
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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LIST OF SYMBOLS AND ABBREVIATIONS

English

a	= major semi-axis of Clarke Spheriod of 1866-6378206.4 meters (section 3.1)
a	= length of geodesic between P_1 and left landmark (section 4.2.4)
a_{ij}	= element in the i^{th} row and j^{th} column of matrix $\underline{A}$
A	= summation notation (appendix B)
A_1	= geometry factor for transformation from measurement space to positioning space
$Area_\sigma$	= area of confidence ellipse - planning
$Area_s$	= area of confidence ellipse - positioning
A_σ	= major semi-axis of confidence ellipse - planning
A_s	= major semi-axis of confidence ellipse - positioning
$A_{\sigma\delta}$	= semi-diameter in direction δ of confidence ellipse - planning
$A_{s\delta}$	= semi-diameter in direction δ of confidence ellipse - positioning
AP	= (assummed, assigned, designated, desired) position
AT	= reference position of the angle takers when measuring horizontal angles
$\underline{A}$	= (nx2) matrix of linearization coefficients (partial derivative of measurements w.r.t. x and y)
b	= minor semi-axis of Clarke Spheroid of 1866, 6356583.8 meters (section 3.1)
b	= length of geodesic between P_1 and right landmark (section 4.2.4) (approximated by chord length for gradient calculation)
B	= summation notation (appendix B)
B_1	= geometry factor for transformation from measurement space to positioning space
B_σ	= minor semi-axis of confidence ellipse - planning
B_s	= minor semi-axis of confidence ellipse - positioning
c	= velocity of electromagnetic radiation
C	= summation notation (appendix B)
COC	= radius of circular region that approximates confidence ellipse and is centered on MPP - positioning
det	= determinant of a matrix (also, vertical bars around argument)
d.f.	= degrees of freedom
D	= summation notation (appendix B)
$\underline{D}$	= length of chord between P_1 and P_2 (appendix A)
$\vec{D}$	= displacement vector from P_1 to P_2
$\vec{D}_p$	= projection of $\vec{D}$ onto plane tangent to spheroid at P_1
$\vec{D}_s$	= AT to chain stopper displacement vector
$\underline{D}$	= (nx1) vector of distance residuals
Diff. per. sec.	= factor to convert Δy in meters to $\Delta\phi$ in seconds
2-drms	= radius of circular region centered on MPP = $2\sqrt{\sigma^2_{maj} + \sigma^2_{min}}$
e^2	= eccentricity of Clarke Spheroid of 1866 = $6.768657997 \times 10^{-3}$
E	= summation notation (appendix B)
F	= summation notation (appendix B)
$F_{n,m}$	= F-distribution with n d.f. in the numerator and m d.f. in the denominator
$F_{n,m,\alpha}$	= F-value of $F_{n,m}$ for confidence level α

G = summation notation (appendix B)
$\underline{G}_i$ = gradient magnitude for i^{th} LOP
$\overline{G}_i$ = gradient vector for i^{th} LOP with direction γ_i
G = (nxn) matrix of gradient magnitudes

$\overline{(G\sigma)^2}$ = one possibility for LOP reference variance
$G\Delta m$ = gradient weighted difference in observed and computed measurements
gwd = sum of the variance weighted gradient weighted differences
H = summation notation (appendix B)
$\underline{H}$ = factor to convert Δx in meters to $\Delta\lambda$ in seconds (appendix C)
$\hat{i}$ = unit vector in positive x direction
$\hat{j}$ = unit vector in positive y direction
$\hat{k}$ = unit vector in positive z direction
k_i = sensitivity factor for position error measure as a function of the i^{th} measurement difference
Δm_i = i^{th} element of $\underline{L} = \alpha_{oi} - \alpha_{ci}$
$\underline{L}$ = (nx1) column matrix of observed and computed measurement differences
$\overline{LOP}_i$ = i^{th} line of position
m = number of lines of position in a subset of a set of n lines of position (appendix E)
m = number of replications (appendix C)
M_s = multiplier for confidence ellipse dimensions - positioning
M_σ = multiplier for confidence ellipse dimensions - planning
MPP = most probable position of sinker drop point
n = number of LOPs or measurements in a positioning scenario
N_1 = radius of curvature of prime vertical at P_1
$\underline{N}_2$ = radius of curvature of prime vertical at P_2
$\overline{O}_i$ = displacement vector from AT to observer taking i^{th} measurement
p = length of geodesic between left and right landmarks (approximated by chord for gradient calculation)
p.d.f. = probability density function
P_1 = point on reference spheroid
P_2 = point on reference spheroid
P-in-R = probability mass contained within a circle of radius R centered on the AP
$\underline{P}$ = (2x2) matrix of eigenvectors used in transformation of $\underline{\Sigma}_x$ to $\underline{\Sigma}_u$
$\overline{r}_i$ = i^{th} element of $\underline{R}$ matrix; residual in i^{th} measurement
r_n = residual in measurement temporarily removed in outlier detection procedure
R = radius of circular region centered on AP for calculation of P-in-R (section 6.2.5)
R = radius of reference sphere used to approximate spheroid (appendix A)
$\underline{R}$ = (nx1) matrix of measurement residuals
$\overline{R}_\alpha$ = radius of curvature in direction α at P_1 on spheroid
R-for-P = radius, R, required to contain at least the probability mass P, as determined at a confidence level, 1-P
s^2 = unbiased estimate of reference variance, σ_0^2
s_{lop}^2 = unbiased estimate of LOP reference variance, $\sigma_{lop_o}^2$
s_R^2 = estimated reference variance when a measurement has been removed in outlier detection procedures
s_{maj} = $A_1 s$
s_{min} = $B_1 s$

S = arc length on reference sphere (appendix A)
S - arbitrary position error measure (section 4.0, appendix D)
S_m = standard established so that $S > S_m$ indicates a successful positioning evolution
SET = systematic error tendency
swd = the sum of the variance weighted measurement differences
t = angle of rotation required to establish a new uncorrelated coordinate axis
$\hat{t}$ = unit vector at AP in cross channel direction, ψ.
TD = time difference
u = abscissa on which major semi-axis is defined
v = ordinate on which minor semi-axis is defined
$\underline{\vec{V}}$ = AP-to-AT displacement vector (before compensation for $\vec{D}_s$)
$\vec{V}_c$ = AP-to-MPP displacement vector (after compensation for $\vec{D}_s$)
$V_{c\psi}$ = component of $\vec{V}_c$ in direction ψ
w_{ii} = i^{th} diagonal element of the $\underline{W}$ matrix, weight of i^{th} LOP (arbitrary units)
w.r.t. = with respect to
$\underline{W}$ = (nxn) weight matrix, elements are $w_{ii} = \dfrac{\sigma_o^2}{\sigma_i^2}$
$\underline{X}$ = (2x1) column matrix of AP-to-AT components
X_1, X_2 = x coordinates of P_1 and P_2, respectively
Δx = constant latitude component of AP-to-AT vector
ΔX = $X_2 - X_1$
Y_1, Y_2 = y coordinates of P_1 and P_2, respectively
Δy = constant longitude component of AP-to-AT vector
ΔY = $Y_2 - Y_1$
Z_1, Z_2 = z coordinates of P_1 an P_2, respectively
ΔZ = $Z_2 - Z_1$

Greek

α = alpha
α = in general, a level of confidence for probability expectations (appendix D)
α = angle between $\vec{D}_p$ and $\hat{\Phi}$ (appendix A)
α_i = i^{th} measurement prior to compensation for $\vec{O}_i$
α_{ci} = measurement computed for i^{th} LOP
α_{oi} = i^{th} measurement corrected for systematic observation errors
β = beta
β = in general, bearing, azimuth or direction w.r.t. (true) north
β_1 = azimuth of left landmark of horizontal angle pair
β_2 = azimuth of right landmark of horizontal angle pair
β_3 = azimuth of right landmark from left landmark of horizontal angle pair
β_{AT} = direction of $\vec{V}$, the AP-to-AT vector
β_{CE} = direction of major semi-axis of confidence ellipse w.r.t. north.
β_h = ships heading
β_m = azimuth of LORAN master station from P_1
β_{MPP} = direction of $\vec{V}_c$, the AP-to-MPP vector
β_{oi} = direction of $\vec{O}_i$ clockwise from β_h
β_s = azimuth of LORAN secondary station from P_1 (section 4.2)
β_s = direction of $\vec{D}_s$ clockwise from β_h (appendix C.3)
γ = gamma

γ_i = positive gradient direction of i^{th} LOP
δ = delta
Δ = upper case delta
$\Delta\alpha$ = conversion (convergence) angle
Δm = difference between observed and computed measurements
δ = chosen direction for calculation of semi-diameter of confidence ellipse
θ = theta
θ_i = azimuth of i^{th} LOP
λ = lambda
$\hat{\lambda}$ = unit vector in direction of increasing longitude
λ_{AT}, λ_{AP}, λ_{MPP} = geographic longitudes of AT, AP, and MPP, respectively
λ_1, λ_2 = geographic longitudes of P_1 and P_2, respectively
$\Delta\lambda$ = $\lambda_2 - \lambda_1$
μ = mu
μ_u = u coordinate of AT in uncorrelated system
μ_v = v coordinate of AT in uncorrelated system
π = pi
π = 3.14159
σ = sigma
σ^2 = in general, the population variance
σ_i^2 = variance of i^{th} measurement
σ_o^2 = arbitrary reference variance
$\sigma_{lop_o}^2$ = arbitrary LOP reference variance
$\sigma_{lop_i}^2$ = variance of LOP_i
σ_{maj}^2 = largest of the two variances of MPP coordinates in uncorrelated system
σ_{min}^2 = smallest of the two variances of MPP coordinates in uncorrelated system
Σ = upper case sigma
Σ = in general, represents a summation operation (appendix B)
$\underline{\Sigma}_h$ = (nxn) diagonal covariance matrix of observations
$\underline{\Sigma}_u$ = (2x2) diagonal covariance matrix of MPP in uncorrelated system
$\underline{\Sigma}_x$ = (2x2) covariance matrix of AT in correlated, (true) north oriented coordinate system
ϕ = phi
$\hat{\phi}$ = unit vector in the direction of increasing latitude
ϕ_{AT}, ϕ_{AP}, ϕ_{MPP} = geographic latitudes of AT, AP and MPP, respectively
ϕ_1, ϕ_2 = geographic latitudes of P_1 and P_2, resectively
$\Delta\phi$ = $\phi_2 - \phi_1$
χ = chi
χ_n^2 = chi-square probability distribution with n d.f.
$\chi_{n,\alpha}^2$ = chi-square value at some confidence level α.
ψ = psi
ψ = chosen direction for calculation of a component of $\vec{V}_c$

1.0 INTRODUCTION

1.1 Background

A need exists to improve the procedures employed by Coast Guard personnel to position aids to navigation. Presently, sophisticated positioning systems such as the Global Positioning System, and Inertial Positioning Systems are not justified for widespread Coast Guard use. Until such systems are justified, alternative procedures must be explored. Presented here are various mathematical, statistical and operational procedures which will be useful in efforts to position aids to navigation both accurately and precisely.

1.2 Discussion of the Problem

A major step taken by the Coast Guard to improve the accuracy in positions of aids to navigation was the commitment made to replace graphical plotting procedures by analytical procedures using horizontal control coordinates and principles of geodesy (reference 20). In this commitment, the Coast Guard displayed an intention to progress from procedures based on common navigational standards to procedures based on the more stringent geodetic survey standards. The expression used herein for the culmination of this progression is "hydrodetic procedures".

Graduation to hydrodetic procedures requires effort on many fronts. Among these efforts are:

a. training of personnel
b. identification of error sources
c. setting standards for aid positions
d. developing procedures to meet the standards
e. developing verification and recording procedures

Employment of hydrodetic procedures can be tedious and is subject to many potential error sources. When computations are performed in the field, this drawback boldly presents itself. Proper implementation of hydrodetic procedures requires planning, operational and administrative procedures. Each is discussed in more detail in the following subsections with reference to similar procedures that are employed in geodetic survey.

1.2.1 Planning Procedures

Implementation of hydrodetic procedures must include analytical planning to define the positioning requirements for each fixed or floating aid. A probability expectation approach is desired to allow designation of the best measurement combinations available in a given situation and the expected result when using any one of the combinations. In addition, the planning process should include identification of the situations that are prone to difficulty and find remedies if possible. The planning phase is of prime importance in positioning aids. In geodetic survey, such phrases as strength of figure, standards of accuracy, general specifications, and recommended spacing are associated with the planning task. In essence, the planning phase is based on the determination of expected accuracy and precision, including propagation of errors. If the proper procedures are followed with

the expected variability, then the position determined should meet the standards specified.

1.2.2 Positioning Procedures

Following planning, the tools required to analytically position an aid are necessary. The adaptation of hydrodetic procedures requires a firm foundation of mathematical models and statistical analysis. Analytical procedures presently being tested need documentation and verification. Guidelines and checks for handling both usual and unusual situations are needed, as are outlier identification procedures. The goal is to provide a consistent set of equations that may be applied at various levels of the aid positioning macro-structure to best achieve the standards established during planning. In geodetic survey, such terms and phrases as base measurement, side check, number of readings, rejection limit and closure are associated with the positioning task. In essence, the positioning phase is based on a statistical evaluation of the accuracy and precision in a set of measurements to see if specified standards have been achieved; and if not, to provide a set of guidelines to achieve them.

1.2.3 Administrative Procedures

For legal and future planning purposes, the data accumulated during any positioning evolution must be verified and recorded. Although this report does not focus on this aspect of the aid positioning problem, it is worthy of mention because planning and operational procedures are dependent upon the constraints imposed by administrative procedures. In geodetic survey, the terms and phrases adjustment, geodetic data publications, horizontal control data and published data are associated with the administrative task. In essence, the administrative phase is one of data accumulation for future use.

1.3 Summary of Previous Work

The capability for using hydrodetic procedures has to some extent been extended to all Coast Guard aid servicing units. The Aids to Navigation Manual - Positioning (CG-222-5) has been revised to include many procedures that can be called hydrodetic procedures. Specifically, creation of gradient diagrams, procedures for selection of landmarks, and collection of horizontal control data are all in progress. These procedures, however, are still far inferior to those employed in geodetic survey. There is no comprehensive set of standards and no analytical capability to measure the adequacy of a positioning effort.

Select ships are involved in providing a data base for use in determining the degree of improvement in accuracy and precision achieved using selected hydrodetic procedures. This effort is documented within work unit 2702.2.2.5 Calculator-Assisted Procedures (CAP) of the ANPAR Project Area (reference 3). A secondary objective of CAP is to create a data base for further study of aid positioning. Feedback collected can be an assist in making an analytical positioning tool which is easy to use.

Past work reported within work unit 2702.1.2 Systems of the ANPAR Project Area has been directed toward literature research, error modeling, and the mathematical concepts, all of which form the basis for this report (references 1 and 2).

1.4 Symbology, Terminology, and Conventions

A list of symbols has been provided and each symbol is defined where first encountered. When possible, the symbols have been selected to agree with previously defined symbology. In some cases, this leads to one symbol representing more than one quantity at different locations in the report. If confusion occurs as to the meaning of a specific symbol, refer to the list of symbols which designates the appropriate meaning for a specific section.

Terminology is in accordance with the Aids to Navigation Manual (CG-222-5) where possible. Terms that might cause confusion are underlined and defined in a glossary. Terms that are defined in Definition of Terms used in Geodetic and Other Surveys, (S.P. #242), U.S. Dept of Commerce, are defined by paraphrasing. An important convention used herein is that all bearings, azimuths and directions are taken clockwise from true North.

2.0 APPROACH - SCOPE

One objective of this report is to develop a comprehensive description for hydrodetic surveys. By its nature, a complete description of a hydrodetic survey requires frequent use of mathematics beyond the level of the average Coast Guardsman. This imposes serious constraints on a logical presentation of a hydrodetic survey from its rudimentary stages to rigorous mathematical detail. The procedures must be immediately accessible to field personnel but, simultaneously, the procedures must be supported by experimental evidence or mathematical derivation. In order to meet the opposing requirements, all mathematics beyond basic algebra have been located within the appendices in the same order as in the body of the report. Frequent reference to the glossary is helpful in understanding mathematical terminology. The following subsections are general descriptions of the major topics discussed in the body and appendices of this report.

2.1 Geodetic Survey

A discussion of geodetic survey, as related to the overall goals of hydrodetic survey, is introduced so that the reader may draw a parallel between hydrodetic and geodetic survey throughout the remainder of the report. It is understood that some of the geodetic survey terminology is introduced prematurely, but the meanings will become apparent as the reader progresses through the parallel hydrodetic survey sections that follow their introduction. Following geodetic survey is a description of the reference system employed in geodetic calculation of distances and directions. Some approximations are made and where appropriate the accuracy of each approximation is discussed. The distances and directions are necessary to both the planning and the positioning sections of this report.

2.2 Planning

The procedures necessary for planning a positioning effort are introduced. These procedures include determination of expected results and suggested procedures to achieve the desired results. The procedures are necessary to define the measurements - and the corresponding lines of position (LOPs) - that might be available to the positioning team when they undertake the positioning effort. Where possible, the procedures are developed so that all user parameters are in dimensions recognizable to the average positioning team member. That is, the standards to be achieved and the measurements needed to achieve them are presented in a specified measurement space for the user.

2.3 Positioning

Equations to compensate for systematic error sources that are known to the user are presented. Equations are derived for the transformation of measurements made by the user into understandable measures of the precision and accuracy of the position determined. An approach which makes use of bivariate statistics is presented for positioning calculations.

2.4 Standards

Section 4.0 conveys a variety of position error measures that could be used for quantitative standards for the accuracy and precision with which the positions of aids to navigation will be established. Standards must give clear guidance to the positioning team, and must also reflect the needs of the users of the waterways. The standards for positions of fixed and floating aids will be different. The results of the positioning effort come in three forms: (1) measures of accuracy, (2) measures of precision, and (3) measures which indicate accuracy and/or precision. The purpose of this report is not to evaluate the various measures but to define them, explain the computations needed to calculate them and discuss any obvious drawbacks in procedures based on their use.

3.0 GEODETIC SURVEY (references 21 and 22)

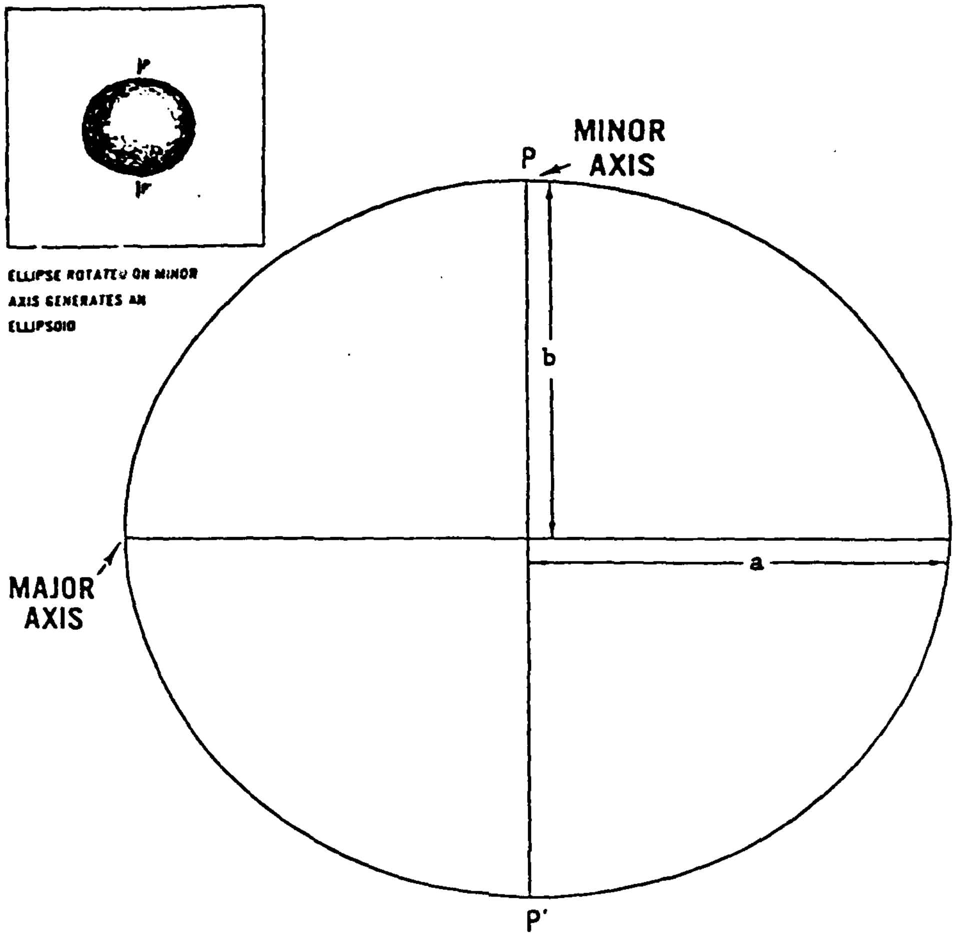
3.1 General

Geodesy may be defined as that science concerned with the exact positioning of points on the surface of the earth and determination of the exact size and shape of the earth. The spherical concept of the shape of the earth offers a simple surface which is mathematically easy to deal with. Many navigational computations use it as a surface representing the earth. While the sphere is a close approximation to the true figure of the earth and satisfactory for many purposes, to the geodesists interested in the measurement of long distances, a more exact figure is necessary. Since the earth is in fact flattened slightly at the poles and bulges somewhat at the equator, the geometrical figure used in geodesy to most nearly approximate the shape of the earth is an ellipsoid of revolution. The ellipsoid of revolution is a figure which would be obtained by rotating an ellipse about its shorter axis (figure 2-1).

Any point on the ellipsoid is designated in terms of latitude, longitude and height, with zero starting references for each. The three values, in addition to the specifications of the ellipsoid itself, are called geodetic datum. Instead of describing the position of a point, P, in terms of latitude, longitude and height as is normally done in geodetic surveying, it can be described by use of a Cartesian coordinate system in x, y and z. The latter is used for equations in this report. The coordinate systems are converted from one to another by mathematical conversion formulae (appendix A.1).

The customary ellipsoidal earth model has its semi-minor axis, b, parallel to the rotational axis of the earth. The size of such an ellipsoid is usually given by the length of the two semi-axes or by the semi-major axis, a, and the eccentricity, e. Any ellipsoid which is very close in shape to a sphere is called a spheroid. The figure of the earth considered as a mean sea-level surface extended continuously through the continents is called the geoid. The ellipsoid (or spheroid) is chosen such that it closely approximates the geoid in the region of interest. Many different ellipsoids have been defined for the various regions of the earth. The ellipsoid used by the Coast Guard and herein is the Clarke Spheroid of 1866 (Ref 6). The Clarke Spheroid has a major semi-axis, a, of 6,378,206.4 meters, a minor semi-axis, b, of 6,356,583.8 meters, and an eccentricity, e, of 8.2271854×10^{-2}. One datum which uses the Clarke Spheroid as the mathematical model of the earth, is North American Datum of 1927 (NAD27) (reference 6). The NAD27 also serves as the datum for nautical charts on which aid positions are depicted.

The primary procedures for positioning buoys for many years have been graphical using the three-arm protractor and charts (details are unimportant here). It will suffice to state that the procedures are inaccurate due to many causes. If positioning an order of magnitude better than that of the navigator is required, then the Coast Guard must complete the conversion to hydrodetic procedures using the NAD27 or an equivalent datum.



a = ONE-HALF OF THE MAJOR AXIS = SEMI-MAJOR AXIS

b = ONE-HALF OF THE MINOR AXIS = SEMI-MINOR AXIS

$$e^2 = \frac{a^2 - b^2}{a^2}$$

FIGURE 2-1. Elements Of An Ellipse

3.2 Survey Procedures

3.2.1 General

In attempting to develop adequate hydrodetic survey procedures, a thorough research of proven geodetic survey procedures is the logical first step. Where possible, direct adoption of applicable geodetic procedures is the most efficient way to reach high standards of accuracy and precision. Unfortunately, such an efficient route is not amenable, except for fixed aids to navigation. Within the U.S. Coast Guard, resection is the primary method of survey upon which hydrodetic procedures are based. Resection is the determination of the horizontal position of a survey station by observed directions from the station to points of known position. The primary source for geodetic survey standards and specifications, reference 25, is void of marine geodetic standards for resection. Standards and general specifications for hydrographic survey are published by the Department of Commerce in the Hydrographic Manual which was prepared by M.J. Umbach in 1976. Unfortunately, these standards and specifications are not directly usable by current Coast Guard positioning teams in their present form. This is primarily due to inadequate surveying expertise and surveying equipment throughout the Coast Guard.

In order for the reader to fully appreciate the complexity of an ordinary acceptable geodetic survey, the following section on selected geodetic survey procedures is provided. As hydrodetic procedures for planning and positioning are introduced, it is worthy to note the frequent parallels with the stages of geodetic survey.

3.2.2 Procedures

Although accuracy and precision are often used interchangeably in everyday conversation, they have distinctly different meanings. Accuracy relates to the quality of a result, and is distinguished from precision which relates to quality of the operation by which the result is obtained. In other words, precision is the degree of refinement in the performance of an operation or in the statement of the result. Precision is of no significance unless accuracy is also obtained.

A primary reference concerned with the accuracy and precision of geodetic survey is the national "Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys," published by the Department of Commerce. The purpose of this section is to discuss the various requirements in that reference.

3.2.2.1 Classification

Surveys are classified:

First Order
Second Order - Class I

Second Order - Class II
Third Order - Class I
Third Order - Class II

with First Order providing the best accuracy and most precision with subsequent orders becoming less accurate and less precise. The standards are provided for three basic methods of survey:

a. triangulation
b. trilateration
c. traverse

Any one could be used in this section but analogies exist such that only triangulation will be discussed.

The surveyor recognizes the existence of error. The procedures specified are directed at eliminating errors or at least reducing the effects of those that are not compensatable. For a survey to qualify as a given order, procedures and results must be both precise and accurate within the specifications. The accuracy of a survey is expressed as the dimensionless quantity, closure. Closure is a ratio that is calculated by comparison of a calculated value against some "true" value. It is not in dimensions of distance. It is based on neither an assumed nor an expected discrepancy, but on an actual comparison with "truth." It is important to clarify the concept of "truth" as it applies to surveying. If any order survey starts from and finishes at higher order stations, the starting and finishing points are considered "truth." "Truth" is often obtained astronomically and in more and more cases, by satellite. If a survey starts and ends from stations of the same order as the survey, those stations are best described as "tentative truth," subject to subsequent adjustment. A survey of a desired order cannot start or end on lower-order stations.

3.2.2.2 Specifications (Triangulation)

The first specification is entitled recommended spacing of principal stations. Higher order stations are generally spaced further apart than lower order stations. The data in any local survey is carried through the principal stations to the global survey network of which the principal station must be a part. In order to conduct an accurate survey, it is important to tie into the global survey network at least as often as the recommended spacing.

The second specification is entitled strength of figure. To avoid a complete discourse on this specification, it is basically an exponential figure-of-merit representing the precision of computed lengths in a triangulation net as determined by the size of the angles, the number of conditions to be satisifed and the distribution of baselines and points of fixed position. The specifications place desirable limits of the strength-of-figure factors for each triangle in the survey chain. Since the measure is exponential in nature, the propagation of error through a chain of triangles is additive. Desirable and maximum limits are placed on strength-of-figure before it is necessary to tie into an existing baseline or to measure one.

The two specifications above are applicable to the planning of the survey. They are based on a measure of expected accuracy and precision, including propagation of errors. The upper limits imposed on individual triangles and on the chain are determined by the ultimate accuracy which must be achieved.

The third specification is the first applicable to actual conduct of a survey. This is entitled base measurement and requires that the starting baseline (and any other baseline which is calculated from the points being surveyed) be measured in length by acceptable procedures so that the standard error of the multiple measurements meet specified standards. Base measurement also provides a validity check for the baseline.

The fourth specification is horizontal directions (or angles) which specify the type of instrument, the number of observations, and the rejection limits for those observations. For example, First Order requires a 0.2" instrument and 16 positions with a rejection limit of 4 seconds from the mean. A position constitutes a direct and a reverse measurement between the two sighted points. This entails four pointings and readings per position (total of 64 for First Order). Each position entails the determination of the mean angle taken from the direct and reverse readings (for First Order, a total of 32 angles).

The fifth specification, triangle closure, consists of two parts: They are "average, not to exceed" and "maximum, seldom to exceed." The three angles measured in each triangle are added together. The difference between their total and 180 degrees plus spherical excess, is the triangle error of closure. Since the triangle cannot be uniquely defined if misclosure exists, it is removed by applying it in equal portions to all angles (residual error averaging). If misclosure is excessive, the measurements are repeated.

The sixth specification is entitled side checks and concerns sides that are common to two different triangulation chains. Side equations are used to ensure that the length and direction are the same using either chain.

The seventh specification is entitled astro azimuths and specifies the frequency (figure spacing) at which astronomical sightings must be made. Requirements are imposed on astronomical sightings that are similar to those imposed on horizontal sightings.

The eighth specification is vertical angle observations and sets forth the number of vertical observations the allowable spread and the number of figures allowed between known elevations. This specification is needed to reduce horizontal control data to the reference ellipsoid.

3.2.2.3 Accuracy

The final requirement is that of closure in length and position. Having satisfactorily met the first eight specifications, this specification concerns the comparison of computed lengths of bases with their measured lengths or with known lengths from finishing on existing control or the comparison of computed positions with their known positions. This misclosure, if within tolerances specified, indicates the achievement of desired accuracy at the specified endpoints; having also met the stringent procedural and variability contraints from start to finish.

3.2.2.4 Summary

a. Instruments used must meet rigid specifications on inherent accuracy, precision and resolution.

b. Procedures specified for use of the instruments involve the averaging of multiple readings to obtain a final value, provide a means of averaging out systematic errors and force the standard errors in the measurement to be small and within calculable limits through the inclusion of rejection limits.

c. Residual error averaging is used, where residuals are acceptably small, to ensure consistency of figures.

d. Propagation of errors is controlled by frequent validation against "truth" (azimuth and baseline).

e. Accuracy is not assumed until misclosure to truth is adequately small, precise procedures have been followed, and variability limitations at all levels have been met.

f. The geometry of a survey is planned such that positive results can be expected.

g. Lease squares adjustment is performed after completion of steps a. through f. above.

3.3 Geodesics

A geodesic is the line of shortest distance between any two points on any mathematically defined surface. Both planning and positioning require accurate calculation of the length and azimuth of geodesics between points on the reference spheroid. The reason for this requirement will become evident in later sections where lines of position are determined. Procedures for length and azimuth computation are readily available in the literature (references 8, 23, and 24 to name a few). The computations can be simplified by using various approximations in the calculations.

Three procedures for approximating geodesic length, one procedure for approximating geodetic azimuth, and the accuracy of each approximation are presented herein. The first approximation to the length of the geodesic is made by approximating the shape of the reference spheroid by a sphere; the

radius of which is calculated by the arithmetic average of the radii of curvature of the geodesic at the endpoints. The length of the chord between the two endpoints of the geodesic is calculated and used to find the length of the great circle arc that connects the endpoints and lies on the approximating sphere.

The second approximation to the length of the geodesic is made using only the endpoint from which the survey originates to define the radius of curvature of the geodesic. The remaining calculations are identical to those in the first approximation described above.

The third and final approximation to the length of the geodesic is made by calculation of the chord length between the two endpoints of the geodesic.

The accuracy of the approximations decrease from the first to the third approximation and is dependent on geodesic length, azimuth and latitude in each case.

The azimuth of the geodesic at the point of interest is approximated by calculating the angle between the chord and the meridian (line of constant longitude) as projected onto a plane tangent (touching only at the point of interest) to the reference spheroid. The forward azimuth is of the geodesic at the survey's origin and the back azimuth is of the geodesic at the opposite end of the geodesic (see figure 3-2). For short geodesics, the forward and back azimuths do not differ significantly from 180°. However, the difference between forward and back azimuth for long geodesics can differ significantly from 180° due to convergence of the meridians. The amount it differs from 180° is called convergence (also conversion angle by navigators).

The following sections provide additional detail on each approximation and provide a description of the accuracy of each. The mathematical details are contained in appendix A-1.

3.3.1 Approximating Length of Geodesics

3.3.1.1 Approximation GL2

The procedure for approximating geodesic length using both endpoints of the geodesic is as follows:

a. define reference spheroid
b. define geodetic coordinates of endpoints of geodesic
c. transform geodetic coordinates of endpoints to Cartesian coordinates
d. calculate length of chord connecting endpoints found in (c.) above by using Pythagorean relation

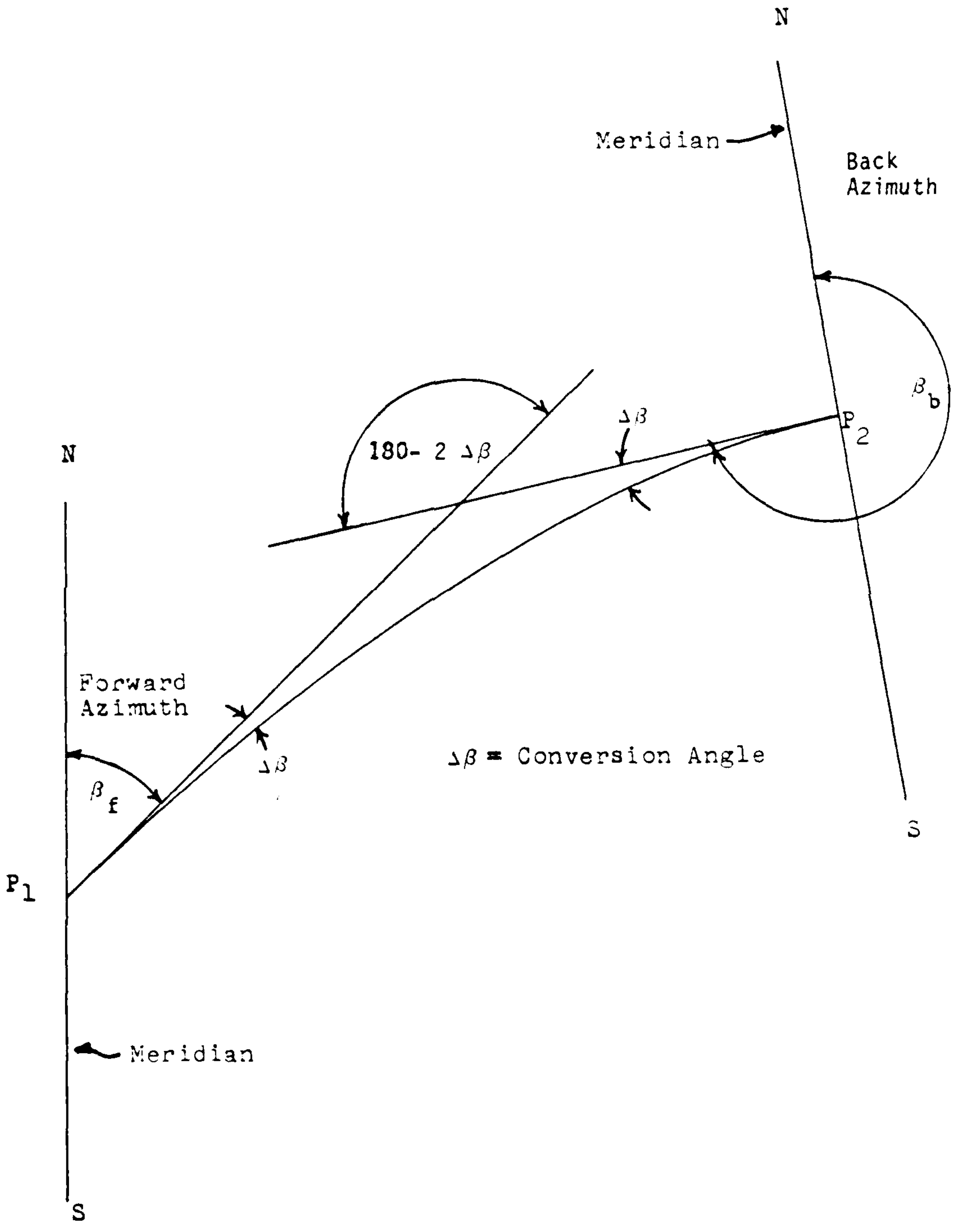


FIGURE 3-2. Azimuths and Convergence

e. calculate radius of curvature of geodesic at both endpoints and find arithmetic average
f. define approximating sphere with radius equal to radius of curvature of geodesic found in (e.)
g. calculate length of arc on sphere defined in (f.) and subtended by the chord defined in (d.)

The mathematical details of this set of calculations are found in appendix A-1. Example results of geodesic length approximations are presented in table 3-1 at two latitudes (0°N, 45°N) and fifteen geodesic length and azimuth combinations.

From the table, it is evident that approximation GL2 exceeds all accuracy requirements on geodesic length for aid positioning operations. For distances greater than 1000 kilometers, an accuracy of within ten meters can be expected. Within normal positioning scenarios (geodesics less than 20 km) the accuracy is within millimeters.

3.3.1.2 Approximation GL1

The steps in approximating geodesic length using one endpoint are as follows:

a. define reference spheroid
b. define geodetic coordinates of endpoints of geodesic
c. transform geodetic coordinates of endpoints to Cartesian coordinates
d. calculate length of chord between points
e. calculate radius of curvature of geodesic at endpoint of interest
f. define approximating sphere with a radius equal to the radius of curvature of geodesic found in (e.)
g. calculate length of arc on sphere defined in (f.) and subtended by the chord defined in (d.)

The mathematical details of this set of calculations are found in appendix A-1. Example results of geodesic length approximations are presented in table 3-1 at two latitudes (0°N, 45°N) and fifteen geodesic length and azimuth combinations.

From the table, it is evident that approximation GL1 exceeds all accuracy requirements on approximating geodesic lengths in aid positioning. For distances greater than 1000 kilometers an accuracy of within 40 meters can be expected. Within normal positoning scenarios the accuracy is within millimeters.

3.3.1.3 Approximation GLC

The procedure for approximating geodesic length by chord length is as follows:

Table 3-1

ERRORS IN APPROXIMATIONS TO GEODESIC
LENGTH AND AZIMUTH

Lat	Δφ	Δλ	GL2	Acc	GL1	Acc	GLC	Acc	GA(P_1)	GA(P_2)	Diff -180°	CA	Acc
	ddmmss	ddmmss	meters	cm	meters	meters	meters	meters	ddmmss	ddmmss	ddmmss	ddmmss	secs
0°	0	0°00'06"	185.54	0	185.54	0	185.54	0	270°00'00"	90°00'00"	0	0	0
0	0	0°01'00"	1855.40	0	1855.40	0	1855.40	0	270°00'00"	90°00'00"	0	0	0
0	0	0°10'00"	18553.98	0	18553.98	0	18553.97	2×10^{-2}	270°00'00"	90°00'00"	0	0	0
0	0	1°00'00"	111323.87	±1	111323.87	0	111322.46	2	270°00'00"	90°00'00"	0	0	0
0	0	10°00'00"	1113238.72	±1	1113238.72	0	1111326.29	2×10^{3}	270°00'00"	90°00'00"	0	0	0
0	0°0'00"	0	184.29	0	184.29	0	184.29	0	0°00'00"	180°00'00"	0	0	0
0	0°01'00"	0	1842.92	0	1842.92	0	1842.92	0	0°00'00"	180°00'00"	0	0	0
0	0°10'00"	0	18429.25	0	18429.25	0	18429.24	2×10^{-2}	0°00'00"	180°00'00"	0	0	0
0	1°00'00"	0	110575.59	±1	110575.57	3×10^{-2}	110574.19	2	0°00'00"	180°00'00"	0	0	0
0	10°00'00"	0	1105867.16	±10	1105848.72	30	1104464.23	2×10^{3}	0°00'00"	180°00'00"	0	0	0
0	0°00'06"	0°00'06"	261.51	0	261.51	0	261.51	0	314°48'24.3"	134°48'24.3"	0	0	0
0	0°01'00"	0°01'00"	2615.12	0	2615.12	0	2615.12	0	314°48'24.3"	134°48'24.3"	0	0	0
0	0°10'00"	0°10'00"	26151.22	0	26151.22	0	26151.20	3×10^{-2}	314°48'24.8"	134°48'23.9"	9.0"	9.0"	0
0	1°00'00"	10°00'00"	156903.52	±1	156903.50	3×10^{-2}	156899.54	5	314°48'40.0"	134°48'08.6"	31.4"	31.4"	0
0	10°00'00"	10°00'00"	1565148.41	±10	1565122.60	40	1561199.00	5×10^{3}	315°14'43.2"	134°22'05.6"	52'37.6"	52'29.6"	8"
45	0	1°00'06"	131.42	0	131.42	0	131.42	0	270°00'02.3"	89°59'58.1"	04.2"	4.2"	0
45	0	0°01'00"	1314.17	0	1314.17	0	1314.17	0	270°00'21.2"	89°59'38.8"	42.4"	42.4"	0
45	0	0°10'00"	13141.75	0	13141.75	0	13141.75	0	270°03'32.1"	89°56'27.9"	7'04.3"	7'04.3"	0
45	0	1°00'00"	78850.00	±1	78850.00	0	78849.50	2	270°21'12.8"	89°38'47.2"	42'25.6"	42'25.6"	0
45	0	10°00'00"	788003.92	±1	788003.90	3×10^{-2}	787504.55	6×10^{2}	273 32 24.1	86 27 35.9	07°04'48.2"	7°09'15.8"	32"
45	0°00'06"	0	185.22	0	185.22	0	185.22	0	0°00'00"	180°00'00"	0	0	0
45	0°01'00"	0	1852.26	0	1852.26	0	1852.26	0	0°00'00"	180°00'00"	0	0	0
45	0°10'00"	0	18522.83	0	18522.83	0	18522.82	.1	0°00'00"	180°00'00"	0	0	0
45	1°00'00"	0	111145.16	±1	111145.15	2×10^{-2}	111143.75	2	0°00'00"	180°00'00"	0	0	0
45	10°00'00"	0	1112325.85	±10	1112318.75	8	1110914.55	2×10^{3}	0°00'00"	180°00'00"	0	0	0
45	0°00'06"	0°00'06"	227.11	0	227.11	0	227.11	0	324°38'42.6"	144°38'38.3"	04.3"	4.3"	0
45	0°01'00"	0°01'00"	2270.99	0	2270.99	0	2270.99	0	324°39'15.0"	144°38'32.6"	42.4"	42.4"	0
45	0°10'00"	0°10'00"	22700.21	0	22700.21	0	22700.20	2×10^{-2}	324°44'34.6"	144°37'29.7"	7'04.9"	7'04.9"	0
45	1°00'00"	1°00'00"	135874.14	±1	135874.13	2×10^{-2}	135871.57	4	325°14'21.5"	144°31'33.7"	42'47.8"	42'47.8"	0
45	10°00'00"	10°00'00"	1320489.45	±10	1320481.33	9	1318132.40	3×10^{3}	330°56'40.7"	143°14'49.0"	07°41'51.7"	7°41 22.9	29"

a. define reference spheroid
b. define geodetic coordinates of points at end of geodesic of interest
c. transform geodesic coordinates of points to Cartesian coordinates
d. calculate length of chord between points

The mathematical details of this set of calculations are found in appendix A-1. Example results of geodesic length approximations are presented in table 3-1 at two latitudes (0^{o}N, 45^{o}N) and fifteen geodesic length and azimuth combinations.

From the table, it is evident that approximation GLC exceeds all accuracy requirements on geodesic lengths for normal positioning problems.

The accuracy of approximation GLC is within centimeters for geodesics as long as 20 kilometers; those that are normally encountered in positioning aids to navigation. However, its use for geodesics of greater than 100 kilometers can cause errors as large as five meters. GLC accuracy is unacceptable for long geodesics (1000 km) where the approximation is only within thousands of meters of the correct length.

3.3.2 Approximating Azimuth of Geodesics (GAC)

The procedure for approximating geodesic azimuth is as follows:

a. define reference spheroid
b. define geodetic coordinates of points at ends of geodesic of interest
c. transform geodetic cordinates of points to Cartesian coordinates
d. calculate length and three-dimensional direction of chord
e. calculate length and direction of projection of chord onto plane tangent to spheroid at point where azimuth of geodesic is to be calculated
f. define direction of meridian on tangent plane to the north
g. calculate difference between direction of meridian and direction of the projected chord found in (e.)
h. convert difference angle to azimuth

The azimuth at the opposite end of the geodesic can be approximated in an identical manner but with the endpoint coordinates interchanged.

The mathematical details of azimuth approximation are contained in appendix A-1. Examples of the accuracy of the azimuth approximations are displayed in table 3-1 under various conditions. To check the accuracy of azimuth calculations under various conditions, the forward and back azimuths were approximated by approximation GAC and their difference was compared to the exact conversion angles using a precise formula. The conversion angles agree to within one half of a minute for geodesics less than 1500

kilometers in length and are exact to a tenth of a second for lengths up to 150 kilometers. This accuracy exceeds any requirements on normal aid positioning.

3.3.3 Accuracy of Approximations

Any of the three geodesic length approximations are more than adequate for computations normally used in positioning aids to navigation, where chord lengths infrequently exceed 20 kilometers.

4.0 PLANNING

Planning must be performed prior to the execution of positioning effort to establish authoritative standards and to ensure the best possible combination of measurements is selected. The planner should study all available measurement combinations mathematically and specify reasonably achievable standards applicable to each aid location. For example, it would be of no use to plan a positioning effort by resection methods if the nearest landmarks are 100 kilometers away. Likewise, it would not be wise to set low standards for aids that mark dangerous areas.

The remainder of this section is divided into two vastly different parts. The first part relates to general requirements prior to positioning and the second concerns calculation of lines of position.

4.1 Requirements Prior to Aid Positioning

The objectives prior to aid positioning are to: (1) identify the measurement combinations which are predicted to provide the best aid positioning data (e.g., accuracy and precision), (2) to project expected results of a positioning effort using the selected combinations, and (3) to provide alternative procedures in the event problems are encountered.

In order to accomplish the first objective, the positioning process is mathematically modeled using assumed measurement variances (appendix B). The result is a planning model. The planner then decides which position error measures are most critical to the aid location. These measures may be of accuracy and/or precision. If the aid is to mark a very distinct obstacle in a channel, then it would be wise to make every effort to place the aid with a high probability of marking that obstacle, i.e., accurately and precisely. If the aid only marks the side of a channel, then reasonable error in the direction of traffic flow may be acceptable.

Let the position error measure be called S. Assume that there are many available measurement combinations at a particular station. By assuming measurement variances, expected values of S can be calculated using the planning model. The measurement combination that provides the best position error measures, S, is the first priority for use in positioning. A priority listing of the combinations can be created for each station.

Objective (2) above can be achieved by a thorough evaluation of the circumstances due to the location of the aid. The planner must consider signal availability and quality, the importance or criticality of the aid, water depth, watch circle and past history of the aid. With all of this in mind, the planner can establish practical standards for the aid. Objective (3) can be met partially by use of the planning model. Assuming that the expected results are not achieved on the first try, alternative procedures should suggest which may include (but certainly are not limited to): checking instruments for error, making redundant measurements, checking for blunders, redefining geometry to the next on the priority list, checking landmarks, adding measurements or simply repeating the entire positioning evolution. The planner must be made aware of the problems incurred so that future planning can solve the problems or avoid similar occurrences in later positioning

efforts. The planning model should be employed to determine which of the above procedures makes the most significant improvement in position error. For example: If a measurement is to be repeated, which one should be repeated?

4.2 Lines of Position

Appendices B and C provide a mathematical discussion of planning and positioning models. The starting point of each appendix is a set of lines of position (LOP) and the gradient vector of each. The gradient vector is defined by the transverse displacement of the respective line of position per unit positive change in the measurement used to define the line of position and is in the direction of the transverse displacement. The differences between observed measurements and the measurements expected at the designated position are also needed for positioning.

The procedures discussed in section 3.3 are used to compute inverses, from which values of measurements that determine lines of position which pass through the designated position are obtained. To accomplish this aim, the following data are required:

a. geographic coordinates of designated position
b. geographic coordinates of observable landmarks or other horizontal control

Performing the computations to find expected values, prior to making actual observations, is called precomputation.

The discussion of measurement precomputation is grouped within four categories. They are:

Category	Type LOP	Example Intrument
time difference (range)	small circle	radar
bearing	great ellipse	gyrocompass
time difference	hyperbolic	LORAN
horizontal (difference) angle	small circle	sextant

The mathematical model is linearized so each conic section (circle, ellipse, hyperbola) is assumed straight in the area of interest.

4.2.1 Range by Time Difference Measurement

The expected range is the length of the geodesic between the desired geographic location, P_1, and the geographic location of the landmark, P_2. The angle, θ, with which the line of position crosses the meridian, passing through the desired position, is found from the azimuth β, of the geodesic at P_1: (figure 3-3)

$$\theta = \begin{cases} \beta + 90 & 0 \leq \beta < 90 \\ \beta - 90 & 90 \leq \beta \leq 270 \\ \beta + 90 & 270 < \beta < 360 \end{cases} \tag{4-1}$$

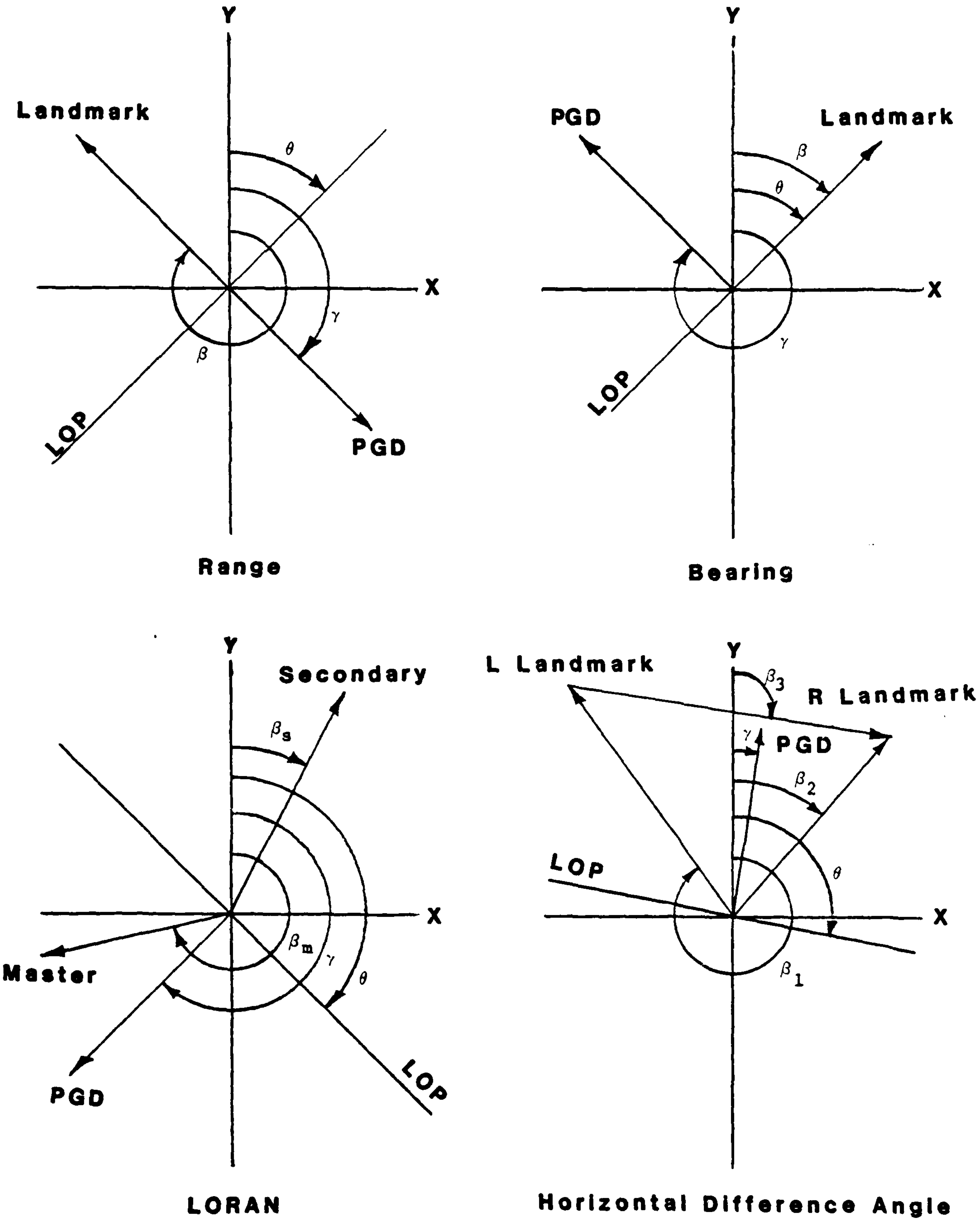


Figure 3-3 POSITIVE GRADIENT DIRECTIONS (PGD)

The gradient magnitude of any range line of position is c/2, where c is the velocity of light in meters per second (EM waves). That is, the LOP is displaced c/2 meters transversely for a one second difference in total time of travel to and from the landmark. Of course, range measurement is not in seconds. Seconds are converted to meters for instrument readout. The gradient magnitude, in terms of the readout, is always one (1) meter per meter difference provided there are no systematic errors. The direction, γ, of the gradient vector is determined from the azimuth of the geodesic at P_1, by,

$$\gamma = \beta + 180^o \qquad 0^o \leq \gamma < 360^o \qquad (4\text{-}2)$$

4.2.2 Lines of Position by Bearing Measurement

The expected bearing measurement is the azimuth of the geodesic at the desired geographic location. The angle, θ, with which the line of position crosses the meridian passing through the desired location is identical to the forward azimuth, β. Because bearing is normally taken in units of degrees, the gradient magnitude is

$$G\left(\frac{m}{deg}\right) = \frac{\pi S}{180^o}$$

where S is the length of the geodesic in meters. The positive gradient direction is determined using the azimuth of the geodesic at P_1 by: (figure 3-3)

$$\gamma = \begin{cases} \beta + 270 & 0^o \leq \beta < 90^o \\ \beta - 90 & 90^o \leq \beta < 360^o \end{cases} \qquad (4\text{-}3)$$

4.2.3 Hyperbolic Lines of Position by Time Difference Measurement

Time difference (TD) measurements can provide the hyperbolic loci of constant measurement points, as typified by LORAN (references 10, 11). The use of LORAN in positioning aids is generally confined to a backup for periods of adverse weather conditions. After calibration and control procedures have been performed, buoy positioning in the repeatable, augmented or hybrid modes can be acceptable. (Presently, LORAN use for aid positioning is never acceptable without calibration and rarely acceptable even with calibration). Each calibration procedure (mode) is described adequately in references 4 and 9 and will only be described briefly here. It is sufficient to discuss LORAN as it relates to precomputations compatible with this report.

The assumption is made for precomputation of TD's that signals propagate along exactly defined paths (geodesics from transmitter to receiver) at a constant velocity. This constant velocity is best estimated by the signal velocity over a salt water path. Of course, errors in precomputation of TD's result as this assumption does not truly reflect actual conditions over land paths or when local anomolies in propagation paths exist. If systematic errors are reasonably constant over an area, the precomputation procedure provides, quite accurately, offsets in time differences between points of known geographic location. The precomputation is based on ground wave and in no way considers the random signal fluctuations in the propagation.

Precomputation of time differences at a desired buoy location vary with the different modes of LORAN operation (reference 9 ch. 7). In the repeatable mode, the precomputed TDs are not used as they are inferior to historical TDs obtained when the aid was positioned by more accurate methods. The augmented mode is an extension of the repeatable mode in which differential corrections to the TDs are observed by another accurately located source in the region and communicated to the positioning team at the time of the positioning effort.

Reference 9 calls for use of a CG computer program to compute TDs when using the hybrid mode of LORAN operation. The TDs are mathematically determined for the desired position and at some other nearby, easy to find, reference point. The vessel visits the reference point, measures TDs, computes corrections and applies them to the predicted TDs for the desired location. Provided that the reference point is easily accessible and no serious local anomolies exist, this procedure can be sufficiently accurate for backup use. The augmented and repeatable modes are preferable, if at all possible.

In cases where the CG Headquarters' computer program is not readily available, an OPM LORAN precomputation capability may prove useful. Of course, the OPM LORAN capability must also be locally calibratable and usable in the quasi-differential or repeatability modes with procedures promulgated in the Aids to Navigation Positioning Manual (CG-222-5). Requirements for such an OPM precomputation capability include the geographic coordinates of the desired position and some preselected reference location in its vicinity, and prerecorded data on geographic location and signals for stations in the applicable chain. An example of the LORAN data required is provided by table 4-1. The table includes the geographic latitude and longitude of each station in the Northeast LORAN-C chain and the respective programmed time delay between the master and secondary signal. The programmed time delays found in the table are slightly different than other published time delays but have been developed through research associated with the CAP effort of ANPAR.

The most efficient use of LORAN-C signals for positioning is to calibrate locally and not even consider the programmed time delays. Calibrating in the local area, in accordance with the ATON Manual, allows determination of the TDs necessary at the desired location.

The gradient vector magnitude of a hyperbolic LOP is provided by, (reference 10)

$$G = \frac{c}{\sin\left(\frac{\beta_m - \beta_s}{2}\right)} \qquad (4\text{-}4)$$

where c is the velocity of electromagnetic waves over sea water and $\beta_m - \beta_s$ is the difference in forward azimuths to the two stations, master and secondary. The LOP direction, θ, is given by: (figure 3-3)

$$\theta = \left(\frac{\beta_m + \beta_s}{2}\right) \qquad (4\text{-}5)$$

with 0° 360°. The positive gradient direction is calculated by,

$$\gamma = \begin{cases} \dfrac{\beta_m + \beta_s}{2} - 90^{\circ} & \beta_m < \beta_s \\ \dfrac{\beta_m + \beta_s}{2} + 90^{\circ} & \beta_m > \beta_s \\ & \beta_m = \beta_s \\ & \text{(depends on position relative to two stations)} \end{cases}$$

Table 4-1

LORAN-C Data Northeast Chain (NAD 27)

	LATITUDE	LONGITUDE	PROGRAMMED DELAY
MASTER	42°.71401944	76°.82623333	-
W	46°.80773889	67°.92754444	13797.20
X	41°.25332778	79°.97791944	26969.93
Y	34°.06266944	77°.91311111	42221.65
Z	39°.85207778	87°.48653055	57162.06

4.2.4 Circle of Position by Horizontal Angle Measurement

The expected horizontal angle measurement is the difference between the forward azimuths of the geodesics connecting the desired geographic location and the landmarks to be observed. The angle, θ, with which the LOP crosses the meridian, passing through the desired location, is determined by: (reference 9) (figure 3-3)

$$\theta = \beta_1 + \beta_2 - \beta_3 \pm 180^\circ \qquad 0^\circ \leq \theta < 360^\circ \tag{4-6}$$

where β_1 is the forward or back azimuth of the geodesic between the desired geographic location and the left most landmark, β_2 is the same measure but for the other landmark, and β_3 is the forward or back azimuth of the geodesic between the landmarks. It is noted that the derivation of equation (4-6) in reference 9 is an approximation. The LOP angle, θ, is found on a plane which approximates the spheroid in the region of interest. The result of this approximation is an incorrect LOP angle. The inaccuracy, however, has a negligible effect on later derivations.

The gradient vector magnitude is determined from the linearized formula: (reference 9)

$$G\left(\frac{\text{meters}}{\text{minute}}\right) = \frac{\pi\, ab}{(180 \times 60')p} \tag{4-7}$$

where a is the chord length approximation of the geodesic between the desired position and the left-most landmark, b is the same approximation to the other landmark, and p is the same approximation between the two landmarks. The computational errors will not significantly affect the results of further derivations. The positive gradient direction, γ, of the line of position is determined from the LOP angle, θ, and the azimuths of the two observed landmarks by

$$\gamma = \begin{cases} \theta + 90^\circ & \beta_1 \neq \beta_2 \\ \theta \pm 90^\circ & \beta_1 = \beta_2 \end{cases} \qquad 0^\circ \leq \theta < 360^\circ \tag{4-8}$$

5.0 POSITIONING

This section is divided into two parts: (1) systematic error and (2) accuracy and precision. The first part concerns error sources for which compensation procedures are either provided here or readily available in other references. Once again, the mathematics are left to the appendices. The second portion puts accuracy and precision into perspective for use in this report.

5.1 Systematic Errors

Compensation for systematic errors is a necessary part in accurate positioning and can take place before or after computations to determine a position, depending on the nature of the error. Compensation for systematic observation errors take place before computations, whereas compensation for eccentrics (translational offsets), such as that induced when the measurement observers are not located at the chain stopper, can be performed after the computations of the position of the angle takers. Systematic observation errors such as sextant error, radar error, gyro error, personal error and inclined angle error are compensated for through procedures readily available in references 4 and 9.

5.1.1 Observer to Chain Stopper Vector

Observers are prohibited from standing near the sinker drop point during positioning operations. The displacement from the reference position of angle takers, AT, to the sinker drop point may be compensated for after determination of AT. Both the direction and distance from the sinker release point to AT are required. The mathematical equations for this translation are found in section C.3. The sinker release point is of prime importance as it represents the MPP. The AP-to-MPP distance is determined from the AP-to-AT distance by applying this eccentric correction either with the mathematical equations or by placing a scaled model of the positioning unit on the gradient diagram.

5.1.2 Lack of Observer Coincidence

It is desirable for observers to stand at the same point when making measurements. Of course, this is difficult due to intervisibility conditions and often a systematic error from observer lack-of-coincidence exists. The distance and direction of each observer from the reference position of angle taker, AT, are used to compensate for this sytematic error. The mathematical equations for compensation are found in section C.2.

5.2 Accuracy and Precision in Positioning

When a position is determined, the accuracy is the difference between the actual position and the determined position. High accuracy is present when the difference is small. It is desirable to describe how much the difference is so that a measure of accuracy can be assigned to the position. A more accurate means of positioning is required to make this determination; but such a means is seldom available so accuracy is, in general, immeasurable for usage in aids to navigation positioning. The need remains to describe, as best as possible, the accuracy in a determined position. The best sidestep of this predicament is to employ statistical techniques to describe accuracy in a manner which best suits the needs of this report. For

those needs, accuracy is described harmoniously with precision. The accuracy statistic derived and computed is supplemented by an equally important precision statistic. Mistakes and uncompensated for systematic error in the observations cannot be ignored in computing accuracy and precision statistics and must be detected as often as possible.

The planning methods described earlier are not sufficient for computing the accuracy and precision statistics of a determined position. They provide expectation probabilities rather than actual positioning statistics. The probable measurement error, while indicative of a large sample of error statistics, is not respresentative of a small unique set of actual errors. It ignores the possibility that mistakes might have been made. The methods used in planning, at best, provide some expectation of the precision of a position but do not predict the accuracy of a position.

The statistical approach to positioning is the obvious extension of the probabilistic methods used in planning. The statistical approach is based on the evaluation of each particular position as a separate entity. The statistical evaluation is sensitive to mistakes and uncompensated systematic errors. The statistical approach provides measures of both accuracy and precision in positioning. A statistical model and procedures that provide the accuracy and precision statistics are provided by Rosenblatt (reference 12). The Rosenblatt model and the least squares principle used in the Error Sensitivity Model (ESM) (reference 2), have been extended in appendix C to meet the requirement of this report.

Various computational procedures are available but all can be described in general terms as follows. With respect to a specified "assumed position" (in practice the desired location), each observation defines a line of position. If there were no measurement errors, each LOP would pass through the assumed position with a known direction, i.e., the LOPs would be the precomputed LOPs which define the point. In practice, observed measurements contain some uncompensated error and the LOPs representing the observations will not all pass through the assumed position. In the standard linearized approach for small relative errors, the actual LOPs (based on measurements) are taken to be parallel to the precomputed LOPs and the distance between them, d, is computed by (reference 12),

$$d = G\Delta m \qquad (5\text{-}1)$$

where, G, represents the gradient magnitude of the LOP and Δm represents the small error in measurement.

The degree to which the measurements are consistent with each other and with the precomputed measurements form a sample for analysis of the precision and accuracy of the determined position. With the relative weight of each measurement provided by the type of measurement, the sample can be used to statistically find a position which best represents the actual position and also provides a source for determining the confidence in that best representation. The position calculated is called the reference position of the angle takers (AT). The displacement vector V, from the assumed position to AT, is derived in appendix C. The precision of AT, is derived statistically by study of the measurement residuals. An unbiased estimate, s^2, of the reference measurement variance, σ_0^2, (references. 13 and 14) is made and generalized-T^2 statistics (reference 15) are used to define the confidence region.

6.0 STANDARDS - MEASURING SUCCESS IN POSITIONING

6.1 Standards - General

Quantitative standards should prescribe, in numerical terms, the acceptable degree of uncertainty associated with the positions of aids to navigation. The objective of this section is to discuss the quantitative (numerical) measures of accuracy and/or precision with which the positions of aids to navigation could be established and maintained. Standards must be derived from the quantitative measures so that they will give clear guidance to operating personnel and reflect the needs of the mariner. A standard is defined as an authoritative value, S_m, for comparison with a position error measure, S, to determine if S is acceptable (which indicates that the positioning evolution is complete). S_m may depend on the specific environment at the time of positioning, the availability of measurements, the importance or criticality of the aid, and the difficulty in achieving Sm at the time of positioning. In reality, both S and S_m may be a set of position error measures and standards. The specific combination depends on the aid. Successful aid positioning is determined by both accuracy and precision; which means that the set of S_m values should contain, at least, standards for these two qualities.

Measures of accuracy and precision need not be restricted to use in geometry selection (section 4.1.1) and first time positioning evolutions. Audit of aid to navigation positions also requires some appropriate quantitative measure.

Practically all aid positioning units within the Coast Guard have used point plotting with a three-arm protractor and chart as a prime method to obtain a measure of success with established confidence. This method is gradually being replaced by more sophisticated hydrodetic procedures, some of which are described in Aids to Navigation Manual - Positioning (COMDTINST M16500.1) The equations and procedures within this report allow for creation of more definitive standards suggesting use of grids and/or calculators.

An assortment of position error measures are discussed in the following paragraphs with detailed mathematics left to appendix D. Frequency histograms of the position error measures actually found through research of historical data are also included in the appendix.

6.2 Standards - Specific

6.2.1 A Posteriori Estimate of Reference Measurement Variance

In section 5.2, accuracy and precision were placed in perspective and defined harmoniously. A statistical approach to positioning alluded to the reference measurement variance (abbreviated to reference variance or σ_0^2). The reference variance is an arbitrary constant with arbitrary dimensions. Its square root is referred to as the reference measurement standard deviation or just reference standard deviation, σ_0. The precision with which the AT can be defined is governed by the stochastic nature of the measurements from which the AT was calculated. The stochastic nature (random error) of the instruments causes inconsistency in a set of measurements. The inconsistency is evident in the residuals of a set of

measurements which, in turn, are used to make an unbiased estimate of the reference variance. The estimate is called the "A Posteriori Estimate of the Reference Variance" and is a possible measure for which standards can be set for precision associated with AT. The reference variance estimate, s^2, can in turn, be used as a part in defining a confidence region for AT (described in detail later).

The reference variance might be considered a benchmark to which measurement consistency can be compared. It is determined through analysis of historical data. Its units are considered arbitrary because it can be chosen to represent the measurement variance of any instrument to be used in positioning. Only the relative measurement variances are of importance and they are so only for weighting different measurement types. In the case of horizontal angles with the sextant, its units are minutes squared. Appendix D provides some reference variance estimates that were observed aboard the REDWOOD (WLM-685) during pilot analytical positioning efforts.

6.2.2 A Posteriori Estimate of Reference LOP Variance

A procedure similar to that presented in the last section can be used to make an unbiased estimate of reference LOP variance. LOP variance represents the random error associated with the location of a line of position. The LOP variance is related to the measurement variance through the gradient of the line of position,

$$\sigma_{lop_i}^2 = G_i^2 \, \sigma_i^2$$

where σ_i^2 is the measurement variance.

The estimate of LOP variance for particular measurement sets are computed through use of the distance residuals which are the measurement residuals converted to distance by the gradient. The reference LOP variance being estimated by the distance residuals is an average of the LOP variances of the set of LOPs in the geometry. Similar to reference measurement variance, analysis of historical data is needed to establish reasonable standards for the LOP variance estimate. Some values for this estimate are provided in appendix D from analysis of pilot analytical positioning efforts aboard the REDWOOD (WLM-685).

6.2.3 Confidence Ellipse Parameters

The stochastic nature of the set of measurements used to determine a position is converted to a two-dimensional probability distribution about the AT via a mathematical transformation. The two-dimensional distribution is called a bivariate probability distribution. An important consideration here is that the variances of the distribution in both dimensions are unknown. One of the most important groups of problems in statistics relates to questions concerning the mean (the MPP here) when the variance is unknown. Anderson (reference 15) thoroughly exhausts the subject.

From that work, the generalized-T^2 statistic was taken for use in determining confidence regions with unknown distribution variances. The regions are bounded by ellipses and therefore are named confidence ellipses.

The statistical analysis needed to define confidence regions is found in appendix C. In that appendix, equations for computation of the following confidence ellipse parameters are presented:

a. major semi-axis
b. area
c. semi-diameter in specified direction
d. circle of confidence

All four of the confidence ellipse measures are functions of the estimate of the reference variance, the level of confidence desired in the MPP, the number of measurements used to determine the MPP and the weight and orientation of each line of position. Each confidence ellipse parameter is a potential position error measure for which standards can be established. Confidence limits of 90% have been specified for field use (reference 9).

6.2.4 AP-to-MPP Vector

The AP-to-MPP vector, $\vec{V}_c$, discussed in section 5.1 is a potential measure of success in positioning in tandem with other position error measures. By itself, the vector, like any vector, has two parts: a magnitude and direction.

The magnitude is a measure of the accuracy in the position determined. The measure of accuracy is not absolute because it is not possible to check its correctness (in geodetic land survey this can be performed by sidechecks, final closure, higher order endpoints, baseline measurement, etc.). The measure is of accuracy only in the sense that it defines how close the positioning team thinks the sinker drop point is to the point they are marking. Together with a measure of precision, the vector is the best measure of accuracy available in hydrodetic survey. Some values for the magnitude of the AP-to-AT displacement are provided in appendix D from analysis of pilot analytical positioning efforts aboard the REDWOOD (WLM-685). (NOTE: The AP-to-AT has not been corrected for observer displacement from the sinker drop point.)

The direction of the AP-to-MPP vector is useful in situations where accuracy is important only in specified directions. The component of the AP-to-MPP vector in that direction is the desired position error measure.

6.2.5 P-in-R

There are only a few measures that combine both the accuracy and precision of the MPP. One such measure is the probability that the determined position is somewhere within a designated circular region of radius R centered on the desired position, P-in-R. Calculation of P-in-R is outlined

in appendix C of reference 2 and requires two-dimensional numerical integration, which is very time consuming. Numerical errors related to its calculation and their causes are presented in appendix D.5. The time required to calculate P-in-R on scene using programmable calculators and equations defining regions of applicability are also provided in the appendix.

The use of P-in-R as a measure of success is restricted to well defined probability distribution functions. This means that the random error associated with each measurement must be assumed prior to its computation. In a way, this is a step backwards from the statistical methods promoted by this report. However, its strength as a measure of success could overshadow this drawback.

The bivariate probability density function defined about the MPP using the generalized-T^2 technique is not well defined and further analysis would be necessary to define a measure similar to P-in-R. Although the P-in-R position error measure may be excellent in planning and assessing the success that is thought to have been achieved, it has limited importance in actual positioning efforts.

Values of P-in-R are provided in appendix D from analysis of pilot analytical positioning efforts (observer measurement standard deviation of 5 minutes) aboard the REDWOOD (WLM-685). The radius, R, was set at 10, 15 and 20 meters for the calculations.

6.2.6 R-for-P

The radius of a circular region, centered on the desired position, that contains at least some specified fraction of the probability mass is called R-for-P. Accurate calculation of this measure numerically is beyond the scope of this report. However, a crude but very conservative approximation that never exceeds 10% error (the 10% refers to error in the probability mass enclosed by the radius, R-for-P) is presented in appendix D.6. The R-for-P approximation can be made regardless of whether or not the probability density function is well defined. REDWOOD data was also used to provide some representative values of this position error measure.

6.2.7 Difference Between Actual and Computed Measurements

6.2.7.1 All Measurements

After numerous operations at a given station, high confidence will be attained in the expected or precomputed measurements. In the cases where high confidence exists, the measurements made in positioning can and should be compared with those that are expected. The agreement in this comparison can be measured statistically by comparing the differences to a priori assumed measurement variances. The standard in this case is a dimensionless number taken from statistical tables.

This procedure depends on known probability density function variances and analysis of historical data is required to provide a reasonable assumed measurement variance. This position error measure is called the sum of the squared, weighted differences, swd, and is defined mathematically in appendix D.7. This measure of success combines position

accuracy and precision. Also defined in appendix D.7 is the gradient weighted differences, gwd. This measure of success describes how far the LOPs are from the AP. It has no advantage over swd except that the differences are distance measures. Data on measurement differences and gradient weighted measurement differences are presented in appendix D. The differences are not weighted by assumed variances.

6.2.7.2 One Measurement

One of the most frequently employed procedures in present Coast Guard aid positioning is called the fixed glass procedure. The fixed glass procedure is performed by setting two sextants on precomputed angles and maneuvering the ship until both measurements agree exactly with the prescribed angles. Obviously, this procedure is limited to only two measurements and must be extended to satisfy the requirement for at least three measurements. To do this, an observer quickly measures a third angle, each time a measurement set is desired, thus allowing a check on the measurement set. The problem is to define limits within which the measurement set indicates an acceptable measurement set. The derivation of the relationship between the third measurement difference and resulting position error is found in appendix D.7. The results show the linear relationship that exists between the measurement difference and the major semi-axis of the confidence ellipse. The slope of the linear function is dependent on the confidence level desired, the weight of the third measurement relative to the first two measurements, and the geometry of the fix. An example of this procedure is included with the derivation. Similar calculations can be performed for other position error measures. The procedure is easily adaptable to the presently used grid method (reference 9) with or without using the fixed glass procedure.

6.3 Summary-Combined Standards

Success in positioning must be indicated by measures of both the precision and the accuracy of the resulting position. The position error measures defined previously are listed in table 6-1. The table indicates whether the measures are of accuracy, of precision, or both.

It is not the purpose of this report to designate the measures that are most appropriate for setting standards; however, the following considerations are important:

a. The numerical measure should provide immediate feedback to the positioning team to remove reliance on graphical measures of success.
b. The physical significance of the measure must be easily understood by senior members of the positioning team.
c. The standards set at any specific location must take into account the peculiarities of that location; this implies that all aids are not of equal importance to the mariner.
d. The measure should be easy to advertise to the mariner and easily defended in court; the measure should not be too restrictive on the positioning team.

Table 6-1

POTENTIAL POSITION ERROR MEASURES FOR USE IN SETTING STANDARDS

	Measure of Accuracy	Measure of Precision	Combined Measure of Accuracy and Precision	Applicable to Grid Procedures+	Applicable to Calculator Assisted Procedures
A Posteriori Estimate of Ref. Meas. Variance		X		I	X
A Posteriori Estimate of LOP Variance		X		I	X
Semi-Major Axis of Confidence Ellipse		X		I	X
Semi-Diameter in Specified Direction		X		I	X
Area of Confidence Ellipse		X		I	X
Circle of Confidence		X		I	X
Assumed Position to Most Probable Position Vector	X			X	X
1) Projection	X			X	X
Probability Mass within Region of Radius R			X		X
Radius for Probability Mass P			X	I	X
Difference Between Actual and Desired Measurements			X		
1) sum of weighted differences			X	I	X
2) third measurement difference			X*	X	X

* The third measurement difference is a measure of accuracy only when two of the three measurements are marking before it is used. It is derived with consideration of measurement inconsistency, not accuracy.

\+ I represents indirectly. This indicates that the corresponding measure can be related through the third measurement difference.

7.0 DETECTION OF MEASUREMENT ERRORS AND REJECTION OF MEASUREMENTS

Under the broad concept of error properties of observations, the conventional theory of errors includes blunders or mistakes in addition to random errors and systematic errors. Blunders may be caused by numerous failures to follow prescribed procedures. From a statistical point of view, blunders are observations that cannot be considered as belonging to the distribution in question. They should not be used with the sample. Consequently, measurements should be planned and observational procedures designed to allow for blunder detection and rejection. In practice, there are a variety of ways to detect blunders:

a. Taking multiple measurements and checking for consistency
b. Careful checking of all reading and recording
c. Checking and verifying performance of instruments
d. Vary procedures to get same desired result
e. Applying simple geometric and algebraic checks
f. Note which measurements deviate from the norm by a significant amount

Despite precautions, some blunders may still remain. Their detection and rejection should be carried out according to principles of statistical testing. These principles require a priori knowledge of the distributions of the random variables involved. It is normally assumed that the observations are normally distributed and the variance of each observation type is known. The a posteriori reference variance estimate discussed in section 6.2.1 provides an excellent vehicle for testing a particular measurement set for existence of a blunder. Assuming that a priori knowledge of the expected variance can be established, the reference variance estimate can be tested via the σ^2 distribution (reference 14). The test is outlined in appendix E in three different subsections:

a. A Posterior Estimate of Reference Measurement Variance
b. Measure Combinations
c. Difference Between Actual and Precomputed Measurements

The first section discusses how the estimate can be compared to the a priori known measurement variance determined from historical data. A statistically significant difference leads to suspicion of a blunder.

The second section discusses ways of finding which measurement of a set is the most likely to contain the blunder.

The final section considers another approach; that of statistical analysis of the actual and computed measurement differences. Both blunder detection and rejection are considered using this procedure.

8.0 OPERATIONAL PROCEDURES FOR POSITIONING

The hydrodetic procedures presented can be adopted for Coast Guard use in different ways, achieving desirable results. Two ways are explained in the following sections: graphical procedures and calculator based procedures. No claim is made that one is better than the other and, in fact, a combination of desirable qualities of each may be the best track to follow.

Each section has been divided into six parts: description, accuracy, adaptability, ease of use, feedback, and recording. The description section is self-explanatory. The accuracy section is concerned with the potential accuracy and precision of positions determined using these procedures. The adaptability section considers how compatible the procedures are with present Coast Guard procedures and training levels. The "ease of use" section concerns itself with the technical level required to employ such a procedure and the cumbersomeness of the actual equipment required. The feedback section discusses the time requirement between measurements and comparison of position error to a standard. The recording section involves storage of data accumulated in the positioning effort. These include the measurements, the position error measures, and other data of interest.

8.1 Graphical Procedures - Grid Diagrams

8.1.1 Description

Geographic coordinates of landmarks in the area and the designated position are found through use of procedures presently being employed by the Coast Guard (reference 9). The alternative fix geometries are examined using a chosen position error measure and a priority listing is created for forwarding to the unit. The first priorities are used to develop grid diagrams as is present Coast Guard procedure. On the grid plot are established limits on each line of position which represent the acceptable limits on the measurements used to define the LOP. The lines of position are labeled in measurement units. The limits correspond to the selected set of standards (remember, third measurement differences can be directly related to most position error measures).

The conning officer safely maneuvers his vessel to the location near where the buoy is to be placed. Measurements are read continuously to the conning officer for check against the standard and the grid diagram. The measurements are sounded in angular units or in fractions of a glass depending on the preferred procedure. The grid diagram is drawn so that each line corresponds to the units sounded. The position is approximated on the grid diagram and corrected for eccentric errors by use of a scaled model of the ship on the grid chart. When it is evident that the sinker will drop close enough to the desired location, it is dropped and the final measurements are taken. See appendix A for other hydrodetic procedures to be followed before and during positioning efforts.

8.1.2 Accuracy

This method is quite accurate. The eccentric errors are corrected using scaled models of the positioning unit.

8.1.3 Adaptability

Grid diagrams are being prepared for buoys throughout the Coast Guard. It is not difficult to calculate the maximum acceptable measurement tolerances for the lines of position on the diagram using equations in the appendix of this report. Little additional training is needed to implement this procedure.

8.1.4 Ease of Use

The grid diagram procedure requires very little technical expertise. The computations are performed at operational levels higher than the positioning unit (reference 9). The systematic eccentricity errors do, however, involve a knowledge of both navigation and the grid diagram for proper compensation. The preparation of grid diagrams is a one-time effort and the paperwork is very manageable.

8.1.5 Feedback

The time between the measurement taking and position error determination is dependent only on how quickly LOPs can be plotted on the grid and how quickly eccentricity errors can be compensated for. For typical situations involving three angles, feedback is on the order of 30 to 90 seconds.

8.1.6 Recording

All recording of data is done manually.

8.2 Calculator Based Procedures

8.2.1 Description

Geographic coordinates of the landmarks in the area and the designated position are found through use of procedures presently employed by the Coast Guard. The alternative fix geometries are examined at either the field or district level using a position error measure chosen at either level and a priority listing is created. The first priorities are used in positioning the aid if possible. If not possible, the unit simply employs a fix geometry that is possible.

The ship is safely maneuvered to the area around the desired location where measurements are taken and entered into a programmed calculator. Immediate output of position error and guidance assist is provided by the calculator. The eccentricity error compensation is a part of the calculation. The position error desired (or set of position errors) is compared against the standards and the decision whether or not to drop the sinker is made. The sinker is dropped, final measurements (not restricted to three) are made, and the important information is recorded on magnetic tapes and paper output. As part of the positioning calculations, an outlier detection and rejection routine makes blunder detection realizable. Routine hydrodetic procedures such as discussed in appendix A can be programmed and employed continuously or at will.

8.2.2 Accuracy

The calculator assisted procedure allows use of many measurements simultaneously, compensates for systematic eccentricity errors and detects obvious measurement inconsistencies. It is very accurate.

8.2.3 Adaptability

Very few Coast Guard personnel have any experience with programmable calculators. However, programs created from equations in the appendix require little computer-user interface once the control data has been entered into the calculator. Training requirements for handling the control data, caring for the equipment, and general calculator operation are far greater than those of graphical procedures. The method is adaptable for use by personnel on any size aid to navigation unit.

8.2.4 Ease of Use

State-of-the-art programmable calculators have been built with user definable keys as if designed for this particular application. The technical level required of the operator is higher than with other methods but surely not beyond the capabilities of a high school graduate. Simple operating instructions would make operation no more difficult than normal ship navigation procedures. The hardware required to perform the task must be light, portable and durable. Data loading and recording must be kept to a minimum.

8.2.5 Feedback

The time between the measurement taking and position error determination is dependent on the selected measure of position error. It can vary from seconds to many minutes. The most useful measures are retrievable in seconds ($<$ 7 seconds on the HP-41C by Hewlett Packard).

8.2.6 Recording

Recording of data can be performed immediately upon completion of the operation automatically in hard copy form.

9.0 CONCLUSIONS

a. Analysis of aid positioning data indicates that there is room for improvement in present aid positioning procedures.

b. Present aid positioning efforts can be supplemented by analytical procedures employing either calculators or extended grid techniques.

c. Classifications, standards of accuracy, and specifications for geodetic survey (reference 25) are not applicable to positioning floating aids but procedures are available to achieve the same objectives with a lower degree of accuracy.

d. Planning and positioning models are used for distinctly different reasons but both are necessary to minimize possibility of error either systematic or stochastic in nature.

e. Exact geodetic computations can be approximated with acceptable accuracy for Coast Guard use in analytical positioning.

f. Mixing of measurement types poses no significant problem when using analytical procedures.

g. Detection of measurement outliers is feasible through use of analytical procedures.

h. The "A posteriori estimate of the reference measurement variance" is a useful statistic easily employed to test fix quality through analytical procedures.

i. Many different mathematical measures of position error exist for planning and positioning.

10.0 RECOMMENDATIONS

The following planning should be performed prior to each positioning evolution.

1. Selection of the priority fix geometries and position error measures which provide the best indicators of the accuracy and precision requirements for each aid location.

2. Determination of reasonable standards for the selected position error measures by use of the planning model.

3. Specifications should be stated for the number of measurements needed, the check measurements required, and the angle closures to be performed.

4. Note where problems in positioning might be expected, such as landmark descriptions, landmark orientations, and poor geometries.

5. Updates of control data on each prospective landmark must be performed.

6. Gradient diagrams should be prepared with all standards readily visible in measurement units. Diagrams should specify scale model size to be used as a representation of the positioning unit on the grid diagram. Diagrams should be prepared for all top priority geometries.

The following hydrodetic procedures should be considered as part of the normal positioning routine aboard all positioning units (regardless of whether or not presently used analytical procedures are extended).

1. Horizon closure before and after movements to ensure instrument accuracy.

2. Angle sum (difference) measurement simultaneous with angle measurement when a landmark is used for more than one measurement.

3. Observe only geodetically controlled landmarks (however, the classification is likely to be of little importance).

4. As prescribed in reference 9, routine determination of index error and instrument/observer random error should be performed.

5. Scaled positioning units (on grid diagrams for angle taker to chain stopper compensation) should be used.

6. Remove any reliance on the three-arm protractor and chart for aid positioning.

It is recommended that the following approach be taken with regards to establishment and implementation of standards based on analytical procedures.

1. Where grid diagrams are being used, they should be extended to include the third measurement difference standards discussed in this report.

2. From information provided here and in error modelling, standards on the magnitude of the AP-MPP vector magnitude should be established.

3. When parts 1. and 2. are in force, classification of aid positions should be made by use of the R-for-P position error measure.

In addition to the grid diagram procedures, calculator based procedures should be explored for practical areas of application. The following approach is recommended:

> Program all applicable positioning equations on state-of-the-art programmable calculators. Reinstate CAP-II aboard the REDWOOD (WLM-685) to demonstrate that calculators can be used to ease the burden of aid positioning as well as make the position determined more accurate and precise.

The fear of mathematics and computer science will not be removed unless we subject personnel at all levels to their use. Even if the analytical procedures are placed at levels higher than the field unit, the field unit personnel should have the computer capacity and computing knowledge to understand and check the standards and procedures supplied by higher levels of administration. Many field units have personnel with the mathematical and programming capability to expand on the ideas presented in this report.

The results of the following reports should be reviewed, understood and integrated to compare with original ANPAR objectives and to form recommendations for additional project efforts.

1. Positioning/Error Model-First Interim Report

2. Positioning/Error Model-Error Sensitivity Model-Second Interim Report

3. Positioning/Error Model - Analytical Positioning of Aids to Navigation (This Report)

4. Sinker Drop Error Analysis

5. Off Station Buoy Analysis

6. Watch Circle Analysis (unpublished)

7. Position Accuracy Report (unpublished)

8. Sextant Evaluation Laboratory Report (unpublished)

Institute a class on "The Positioning of Aids to Navigation" at the Coast Guard Training Center, Governors Island. Direct the class toward the junior officer, senior enlisted man, and cadet destined for service on a positioning unit. Structure the course to include:

1. References

   a. ATON Manual - Positioning, COMDTINST M16500.1, 1978
   b. Hydrographic Manual, Umbach, M.J., US Dept of Commerce, 1976
   c. Handbook on Grid Usage (to be published for course)
   d. Chart No. 1 Nautical Charts Symbols and Abbreviations
   e. Pocket Calculator Use in Positioning (to be published for course)
   f. Geodesy for the Layman, US Dept of Commerce, NOAA, NOS, 1977
   g. Sextant Adjustment Manual, CG ATON School, Governor's Island
   h. Classifications, Standards of Accuracy, and General Specifications for Geodetic Control Surveys, FGCC, US Dept of Commerce, 1976.
   i. Specifications to Support Classifications, Standards of Accuracy, and General Specifications for Geodetic Control Surveys, FGCC, US Dept of Commerce, 1975.

2. Course Content

   a. Grid Usage
   b. Calculator Usage → Analytical Procedures
      1. Positioning
      2. Blunder Detection Methods
      3. Methods for correcting poor situations
   c. Survey and Geodesy Basics
   d. Sextant Usage
   e. Discussion of Accuracy and Precision

REFERENCES

1. Millbach, M.A., Positioning/Error Model-First Interim Report, CG R&D Center, May 1979.

2. Millbach, M.A., Positioning/Error Model-Error Sensitivity Model, CG R&D Center, June 1980.

3. Project Plan-ANPAR (2702), CG R&D Center, 1979.

4. American Practical Navigator (Volume I), Defense Mapping Agency, Hydrographic Center, 1977 edition.

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GLOSSARY

Accuracy - Degree of conformity with a standard. Accuracy relates to the quality of a result. The accuracy attained in a positioning effort is a product of the procedures to be followed in executing the work and the precision with which those instructions are followed.

Adjustment - The determination and application of corrections corresponding to the errors affecting the observations, making the observations consistent among themselves, and coordinating and correlating the derived data. Adjustment is commonly performed by the method of least squares.

Administrative Procedures - Accumulation and processing of positioning data for verification, legal and planning purposes.

Analytical Planning - Employment of numerical computations in a systematic procedure for deriving the expected results of a positioning effort and setting standards for the effort.

Astronomical Sighting - Observation of the azimuth of a celestial body.

Audit - Independent check on the position of aids to navigation.

Azimuth - The horizontal direction reckoned clockwise from the meridian plane. In the basic control survey, azimuths are measured clockwise from south. The common procedure for navigation is to measure clockwise from north. North is used in this report.

Azimuth, Astro - The data obtained through an astronomical sighting. At the point of the observation, the angle measured from the vertical plane through the celestial pole to the vertical plane through the observed object.

Azimuth, Forward - For a geodesic line from A to B, the angle between the tangent to the meridian at A and the tangent to the geodesic line at A.

Azimuth, Back - For a geodesic line from A to B, the angle between the tangent to the meridian at B and the tangent to the geodesic line at B.

Baseline - The side of one of a series of connected triangles, the length of which is measured with perscribed accuracy and precision, and from which the lengths of the other triangle sides are obtained by computation.

Base Measurement - Determination of the length of the baseline classified according to the character of the work they are intended to control. Probable error in measurement not to be exceeded for the various classes is prescribed.

Bearing - The horizontal direction of one terrestrial point from another. It is measured from north clockwise through 360°.

Bivariate Statistics - Statistics of, relating to, or involving two variables.

Blunder - A gross error or mistake resulting usually from stupidity, ignorance, or carelessness.

Chord - A straight line segment joining two points on a curve (e.g., circle, sphere, ellipsoid).

Clarke Spheroid of 1866 - See spheroid. The Clarke Spheroid has a major semi-axis of 6,378,206.4 meters and a minor semi-axis of 6,356,583.8 meters.

Class (survey) - The division or rating of surveys based on the overall acc uracy and precision required of the survey.

Closure, Error of - The amount by which a value of a quantity obtained by surveying operations fails to agree with another value of the same quantity held fixed from earlier determinations or with a theoretical value of the quantity. The quantity may be an angular measure, distance measure or spacial coordinates.

Conditions - An equation which expresses exactly certain relationships that must exist among related quantities, which are not independent of one another, exist a priori, and are separate from relationships demanded by observation.

Confidence Ellipse - One of a family of contours of equal probability density of the bivariate probability density function of a position determined by measurements. Two types of ellipses are considered: (1) those which may be expected, based on the laws of chance and the assumed values of measurement random error as used in planning, and (2) those which result by statistical inference from analysis of a particular fix.

Convergence - The angular drawing together of the geographic meridians in passing from the equator to the poles. For a geodesic, the azimuth at one end differs from the azimuth at the other end by 180° plus or minus the amount of the convergence of the meridians at the end points.

Conversion Angle - The angle between the rhumb line and the great circle between two points.

Designated (desired, computed) Position - The point specified by geographic coordinates that authorities have decided an aid is to be placed.

Direct and Reverse Sightings - A method to reduce systematic reading errors by which the telescope is rotated and turned such that the readings are made opposite on the horizontal plate (i.e., 180° different).

Direction - The position of one point in space relative to another without reference to the distance between the points and provided in terms of the angular difference from the reference direction (normally the reference is north).

Direction (survey) - Horizontal angles at a triangulation station are reduced to a common initial and termed horizontal directions.

Distance Angles - An angle in a triangle opposite a side used as a base in the solution of the triangle, or a side whose length is to be determined.

Distance Residuals - The computed distance along a line perpendicular to a line of position between the most probably position of the observers and the line of position.

Eccentricity - the ratio of the distance between the center and the focus of a ellipse to the length of its major semi-axis.

Ellipse - A closed plane curve generated by a point moving in such a way that the sums of its distances from two fixed points is a constant.

Ellipsoid - The three-dimensional figure formed by rotating an ellipse about its major or minor axes.

Error, Modeling - A system of postulates, data, and inferences presented as a mathematical description of errors found in a process.

Error, Propagation of - involves obtaining the stochastic characteristics of (functionally) dependent variables given the characteristics of the independent variables and the functional relationships relating the two sets of variables.

Error, Random - A chance error, unpredictable in magnitude or sign.

Error, Residual (Residuals) - The difference between any value of a quantity in a series of observations, corrected for known systematic errors, and the value of the quantity obtained from the combination or adjustment of that series.

Error, Standard - Sample standard deviation of the mean.

Error, Systematic - An error that is not determined by chance but whose sign and, to some extent, magnitude bear a fixed relation to some condition or set of conditions.

Geodetic Coordinates - Quantities which define the horizontal position of a point on the spheroid of reference with respect to the planes of the geodetic equator (latitude) and of a selected geodetic meridian (longitude).

Geodetic Datum - Numerical or geometrical quantity or set such quantities which may serve as a reference or base for other quantities. Geodetic datum consists of 5 quantities: latitude, longitude, azimuth of some geodesic at the point of concern and the two constants needed to define the terrestrial spheroid.

Geodetic Survey Standards - Quantities specified by the Federal Geodetic Control Commission and published by the U.S. Department of Commerce to ensure high precision and good accuracy in horizontal and vertical control surveys.

Geodesy - The science which treats mathematically the figure and size of the earth.

Geodesic (Geodesic Line) - A line of shortest distance between any two points on any mathematically defined surface.

Geoid - The figure of the earth considered as a mean sea-level surface extended continuously through the continents.

Geometry (of a fix) - Consists of all the lines of position which represent the measurements that compose the fix, their respective measurement standard deviations, and their orientation with respect to each other.

Global Survey Network - Horizontal control survey net: arcs of triangulation, sometimes with lines of traverse, connected together to form a system of loops or circuits extended over an rea. Global indicates that all available triangulation nets are considered.

Gradient Diagram (Grid) - A graphical representation of the computed lines of position that correspond to a specified geometry at an aid location. Lines of position are spaced at regular intervals in measurement units parallel to the desired lines of position, thus forming a grid.

Gradient Vector - Describes quantitively by magnitude and direction the transverse movement of a line of position for a change of one measurement unit.

Graphical Plotting Procedure - In resection, the use of the three-arm protractor and chart in position determination.

Horizontal Control Data - Data established from surveys which are used with measurements to define the position represented by the values of the measurements.

Horizontal Directions - See Directions.

Hydrodetic Procedures - Procedures based on as many geodetic survey standards and methods as possible but which are performed in the marine environment.

Instrument Specification - Indicates the instrument resolution required for conduct of an acceptable survey of a given order and class.

Inverse - The computation of the length and forward and back azimuths of a geodesic by computation based on the known geodetic positions of the ends of the line.

Least Squares Principle - A mathematical method of determining the most probable values of a series of quantities from a set of observations greater in number than are necessary to determine those quantities.

Line of Position - A line on some point of which an observer may be presumed to be located, as a result of observation or measurement.

LOP Variance - The square of the standard deviation of the transverse spacial coordinate of a line of position. The LOP variance is the product of the square of the gradient magnitude and the measurement variance used to determine the line of position.

Lower Order Stations - In a survey, the stations which are not principal stations (less than 3rd order stations).

Major Semi-Axis - The distance from the center of an ellipse to the ellipse along the longer axis of the ellipse.

Marine Geodetic Standards (Hydrodetic Standards) - Standards applicable to survey work at sea.

Marine Geodesy - Geodesy applied to the environment of the sea with intentions of extending geodetic control to that environment.

Measurement Combination - A subset of the measurements made in determining a position.

Measurement Variance - The square of the standard deviation of measurement.

Meridian - A north-south line from which longitudes and azimuths are reckoned; or a plane, normal to the geoid or spheroid defining such a line.

Minor Semi-Axis - The distance from the center of an ellipse to the ellipse along the shorter axis of the ellipse.

Misclosure - See Closure, error of.

NAD 27 - North American Datum of 1927. The geodetic datum which is defined by the geographic position of triangulation station Meades Ranch and the azimuth from that station to station Waldo, on the Clarke Spheroid of 1866.

Number of Positions - Specifies the number of locations on the horizontal circle of a direction theodolite to be used for the observation on the initial station of a series of stations which are to be observed on.

Operational Procedures - Those procedures used in the practical application of the principles and processes of positioning.

Order (Survey) - A category describing the quality of a survey.

Outlier - A measurement that is far from the mean due to chance or blunder.

P-in-R - The probability mass, P, contained by a circular region of radius, R, centered on the desired position.

Planning Model - A mathematical model of the positioning process used to assess the expected results of specific positioning efforts, and to establish standards for positioning, to study problems incurred during positioning efforts.

Position - A location on the horizontal circle of a direction theodolite to be used for the observation of the initial station of a series of stations which are to be observed on.

Position Error Measure - A quantity that represent the accuracy, precision or both accuracy and precision of determined position; to be used in setting standards and as a measure of positioning success.

Precision - The degree of refinement with which an operation is performed or a measurement stated. Usually represented by the standard deviation of a set of measurements of the same quantity.

Prime Vertical - Vertical circle perpendicular to the plane of the celestial meridian. The plane of the prime vertical cuts the horizon in the east and west points.

Principal Station - A station through which basic data are carried in the extension of a survey system. The principal station is a higher order station relative to those stations whose purpose is limited to the control of local surveys.

Probability Expectation Approach - Any approach which uses or assumes a known probability distribution for each step.

Propagation of Error - See Error, propagation of.

Propagation Velocity - The speed at which electromagnetic radiation passes through a medium. Usually termed the speed of light.

Published Data - The geodetic datum of all control stations, formalized for use as a reference base in future surveys.

Radius of Curvature - The reciprocal of the curvature of a line.

Recommended Spacing - The specified distance between principal stations in a survey network.

Reconnaissance - Preliminary survey to gain information and to plan a geodetic survey.

Recording - Record keeping of data for use in planning, research and for legal purposes. Recording includes, but is not limited to, magnetic storage of data.

Reference Measurement Variance - An arbitrary constant with arbitrary dimensions, usually selected as the best known estimate of measurement variance of the observer-instrument combination.

Reference Position of the Angle Takers (AT) - In many cases, measurements can not be taken from the exact same location due to intervisibility conditions. The point selected in the region where measurements are being taken to define displacement vectors for each observer's location is the reference position of the angle takers. When the least squares principle is employed, the actual position determined is AT, and a translation to the point of sinker drop is needed to define the MPP.

Rejection Limit - A specification on the difference between a measurement and the mean of the set of measurements of which it is an element.

Residual Error Averaging - The mathematical operation of spreading a residual error to the measurements from which the residuals are calculated so that the measurement set is consistent with theory.

Resection - The determination of the horizontal position of a survey station by observed directions from the station to points of known position.

Residuals - See error, residual.

Resolution, Instrument - The smallest increment of change that an instrument and observer can measure.

R-for-P - The radius, R, of a circle centered on the desired position that is required to contain, at least, a desired probability mass, P.

Side Check - A specification that requires agreement in triangle side lengths as computed in various chains.

Sphere - The three-dimensional solid figure which consists of the set of all points equidistant from a point constituting its center.

Spherical Excess - The amount by which the sum of the three angles of a triangle on a sphere exceeds 180° (also assumed for spheroids).

Spheroid - Any figure differing but little from a sphere. In geodesy, a mathematical figure closely approximating the geoid in form and size, and used as a surface of reference for geodetic surveys (normally an ellipsoid of revolution).

Standards - An exact value, or concept thereof established by authority, custom, or common consent, to serve as a model or rule in the measurement of quantity, or in the establishment of a practice or procedure.

Standards of Accuracy - Standards established on the degree of perfection obtained in a survey. They serve as benchmarks for determining the quality of a result.

Station - A definite point on the earth, whose location has been determined by surveying methods.

Strength of Figure - Expresses the comparative precision of computed lengths in a triangulation net as determined by the size of the angles, the number of conditions to be satisfied, the distribution of base lines, and lengths determined in previous adjustments. An expression of relative strength.

Systematic Error Tendency - A vector quantity describing how much displacement the position determined by a set of measurements changes when all measurements are changed one unit.

Tangent - Touching a curved surface at only one point.

Traverse - A sequence of lengths and directions of lines between points on the earth, obtained by or from field measurements and used in determining positions of the points.

Triangle Closure - The amount by which the sum of the three observed angles of a triangle fails to equal exactly 180^{0} plus the spherical excess of the triangle.

Triangulation - A method of surveying in which the stations are points on the ground at the vertices of a chain or network of triangles, whose angles are observed instrumentally and whose sides are determined by computation from selected triangle sides called base lines, the lengths of which are obtained from direct measurements on the ground.

Triangulation Net - See Global survey network

Trilateration - A method of extending horizontal control where the sides of triangles are measured rather than the angles as in triangulation.

Truth - Quantity accepted to be perfect. Measurement results are compared to truth to determine accuracy. Truth in survey is provided by higher order stations.

Tentative Truth - Quantities may be determined by higher order procedures and specifications but until they are adjusted to fit the triangulation net, they are only tentative truth.

Verification - Process of checking computations to ensure compliance with applicable standards and freedom of correctable error.

APPENDIX A

MATHEMATICS OF GEODETIC SURVEY

A.1 SURVEY PROCEDURES

Geodetic survey specifications and standards of accuracy are discussed in this appendix. Each section is titled by an appropriate geodetic survey term, those introduced in section 3.2.

A.1.1 Strength of Figure (reference 6)

The strength of figure is derived from that portion of the formula for probable error of a triangle side which is independent of the accuracy of the observations, as follows:

$$\frac{N_d - N_c}{N_d} \sum \left[\delta_A^2 + \delta_A \delta_B + \delta_B^2 \right] \qquad \text{(A-1)}$$

in which N_d and N_c are the numbers of directions observed and of conditions to be satisfied, and δ_A and δ_B are the rates of change in the sines of the distance angles A and B, usually expressed by the differences of the logarithms of the sines for a difference of 1 second in the angles, the sixth decimal being the unit place. By summing up the values obtained by formula for the simple figures composing a triangulation net, the strength of figure for the net can be obtained. As a triangulation net is usually composed of several different systems of simple figures, comparable values of different systems are obtained, and the strongest route can then be selected through which to carry a computation of length. Reconnaissance for a triangulation net is usually executed under instructions which specify limiting values for the strength of figure for the best and second-best chains of triangles between adjacent base lines, the sites for stations and for baselines being selected accordingly.

Strength of figure is not applicable to buoy positioning in the Coast Guard. It is, however, a necessary element of fixed aid position computations. For planning purposes, other figures of merit must be used prior to buoy positioning. The figures of merit suggested in this report are described in Appendix B.

A.1.2 Recommended Spacing of Principal Stations

Principal stations are the link between local geodetic surveys and the global survey network. In geodetic survey the link is made a specified number of times per distance surveyed to ensure accuracy and consistency of the system.

Recommended spacing is not applicable to the CG except when surveying fixed aids to navigation. For floating aids, the link to the geodetic survey network is made through the landmarks used to position the aid. Third-Order Class II or better stations are required for sighting of horizontal angles. Error modeling in reference 2 provided preliminary evidence that large errors (greater than target region dimensions) in a landmark [illegible]

cause significant position error. In cases where insufficient Third-Order Class II or better landmarks are available for sighting, lower order landmarks are acceptable for sighting (with small chance of causing position error) until Third-Order Class II or better landmarks are made available. In no way does sighting Third-Order Class II landmarks allow classification of the surveyed position to be Third-Order Class II.

A.1.3 Base Measurement

The methods of triangulation are quite demanding. They require a considerable number of sightings with a minimum amount of distance measurement. The triangles are developed into a net of interconnected figures, and certain lines, called base lines, must be measured in order to compute the lengths of other sides in the net. The base lines must be measured with extreme precision, since errors propagated through the system originate with those measured lines.

The precision with which a base line must be measured is specified for the various classes of survey by a ratio of standard error of the mean of base line measurements to the length of the base line. The standard error of the mean, σ_m, is computed by:

$$\sigma_m = \sqrt{\frac{\sum v^2}{n(n-1)}} \qquad \text{(A-2)}$$

where v is a difference between a measured length and the mean of all measured lengths, and n is the number of measurements. For Third-Order Class II triangulation, the base measurements must be precise to 1 part in 250,000.

Base measurement is not easily adaptable to buoy positioning but it is, of course, appropriate for fixed-aid positioning. Precise base measurement decreases errors at their origin before they are propagated through the system. A similar goal in resection methods is to decrease the number and size of systematic errors in horizontal angle measurement prior to angle measurement. One way to accomplish this is to close the horizon before and after ship movements. Assuming a priori the random error of a sextant-observer pair acceptable ranges can be computed for horizon closure. The standard deviation of the sum of n angles of equal precision that close the horizon is $\sqrt{n}$ times the standard deviation of each angular measurement.

Ninety-five percent of the sums of n such measurements are expected to fall within two standard deviations of 360° if all measurements are free of systematic errors. It is reasonably safe to assume a systematic error or mistake exists if the sum falls outside of these limits.

A.1.4 Horizontal Directions

The instrument used, the number of positions, and the rejection limit from the mean are considered under horizontal directions.

Resolution is the smallest increment of change that an instrument and observer can detect. For geodetic survey instruments, the resolution is

specified in tenths of seconds for First and Second Order surveys and as one second for Third Order surveys. Normally, sextants are graduated in tenths of minutes. Both systematic and random error of the average sextant are on the order of minutes (reference 30), therefore, specifications for resolution are not critical for sextants and the normal tenth-of-a-minute resolution is adequate. However, it is important to specify acceptable random error in angular measurement. This is already prescribed in reference 9.

The number of positions specification requires the number of sighting groups required for measurement of each angle. This specification requires that the instrument be stationary between measurements which is, of course, impossible when using the sextant aboard a floating unit. Each sighting group requires four separate sightings. The angle must be turned twice and the telescope must be dumped twice for direct and reverse sightings. For Third Order Class II stations two positions require a total of 8 sightings.

Similar redundancy requirements for resection methods can be made by designating the number of simultaneous (or nearly so) measurements required to determine a position. Current requirements (reference 9) are for a minimum of three horizontal angle measurements.

Three measurements provides some redundancy in the position determination but by no means does it allow Third-Order Class II specifications to be achieved. To satisfy Third-Order Class II specifications, eight simultaneous measurements of each of three different horizontal angles would be required by six different 12-man positioning teams (six measurements needed to reach 1-second resolution from 6-second instrument, 24 men to assign four per angle). Thus, 144 simultaneous measurements would be taken. The folly of this process confirms that Coast Guard specifications and standards of accuracy are required; the geodetic survey standards are not achievable from floating platforms.

The rejection limit specification for geodetic survey applies only when many measurements of the same quantity are made at different positions on the horizontal circle and the mean of the measurements at each of the positions is calculated. In turn, the mean of the resulting position means is calculated. Each position mean is compared to the overall mean and if any differs by more than a specified amount from that mean, the set at that position is rejected and repeated. To explain what this accomplishes mathematically, let p represent the number of positions, let r be the rejection limit, let m_i be the mean of measurements at the i^{th} position, let m be the overall mean. The requirement is that:

$$|m_i - m| \leq r \text{ for } i \text{ from } 1 \text{ to } p \tag{A-3}$$

This, in effect, is placing an upper limit on the standard error of the mean. Using equation (A-2) with the maximum acceptable values for differences from the mean:

$$\sigma_m = \sqrt{\frac{pr^2}{p(p-1)}} = \frac{r}{\sqrt{p-1}} \tag{A-4}$$

which equals 5 seconds for Third-Order Class II survey. Detection and rejection methods applicable to this report are found in appendix E.

A.1.5 Triangle Closure

In geodetic survey, whenever a triangle is formed within the triangulation net, the sum of the three angles of the triangle must be within a specified range of 180° plus spherical excess. The triangle closure test is the simplest available in the field to ascertain the accuracy of triangulation observations. There are specifications for both the average and "maximum seldom to exceed" triangle closures for a triangulation net. The effect of this test is the same as that of closing the horizon with horizontal angle measurements.

The triangle closure specification is not adaptable to resection methods. A viable alternative is to measure angles which are geometrically the sum or difference of other simultaneously measured horizontal angles. With an a priori accepted value of measurement standard deviation, the difference between the sum (difference) of two measurements and the measured horizontal angle can be compared statistically as a check for measurement accuracy. The mathematics of this comparison are similar to that of horizon closure. The checking difference between the check measurement and the sum (difference) of the checked measurements should be zero. The standard deviation of the checking difference is the square root of the sum of the variances of the component measurements. The acceptable difference in the check angle should be limited by a multiple of the standard deviation of the checking difference of the component measurements. For example, two measurements are made using a common landmark, each with a standard deviation of five minutes. The check measurement is made with the same precision on the sum (difference) of the two measurements. The checking difference between the sum (difference) of the two measurements and the check measurement should be within 17 minutes ($2 \times 5\sqrt{3}$ minutes) of zero 95 percent of the time. It is safe to question the accuracy of all measurements if this is not the case. Residual error averaging can be employed if the measured angles differ by less than a prescribed standard.

A.1.6 Side Checks

In geodetic survey procedures, side checks are made periodically in the triangulation net as the best check on accuracy in the field. Side checks are comparisons of common sides of triangles determined through different chains in the triangulation net.

The side check is not adaptable to resection methods. A simple, but not strong (reference 2) substitute is to specify limits on angles, bearings, ranges, and time difference measurements as independent checks on the position. The specified acceptable difference in a measurement from that expected should be in a distance dimension. This requires a conversion to measurement units for the instrument used (divide by gradient).

The specified limits are determined by the random error associated with the instrument.

A.1.7 Closure

The accuracy of a fix is the final determination for classification. If all of the standards and specifications are complied with, the final measure of accuracy should be readily attained. The final measure in geodetic survey is the ratio of the error in the final position to the length of a side determined by previously performed higher order surveys. The acceptable ratios have been determined empirically.

For resection on floating platforms the same check with higher order stations is not available (i.e., there is no previously validated station to compare computation results with). An alternative is to compare the determined position to the designated position. The final classification is made by consideration of both the accuracy and precision (defined harmoniously) statistics of the determined position (those discussed in section 6.0, STANDARDS).

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A.2 GEODESICS

A.2.1 Length of Geodesics

The equation of the reference ellipsoid in a Cartesian coordinate system is:

$$x^2 + y^2 + \frac{z^2}{1-e^2} = a^2 \qquad \text{(A-5)}$$

where a is the semi-major axis (center to equator) and e^2 is the square of the eccentricity. Let the positive z-axis pass through the north poles, the positive x-axis pass through 0^o longitude, and the positive y-axis pass through 90^oE longitude. For the Clarke Spheroid of 1866, a = 6,378,206.4 meters and e = 8.2271854×10^{-2}.

A geodesic is the shortest line connecting two points on the spheroid. Define two points $P_1(\phi_1, \lambda_1)$ and $P_2(\phi_2, \lambda_2)$ on the surface of the spheroids. ϕ and λ are the conventional latitudes and longitudes of the points respectively, as they are published on nautical charts and as geodetic control data. They are also called <u>geodetic</u> or <u>geographic coordinates</u>. West longitudes are negative. <u>The azimuth of P_2 from P_1 is called the forward azimuth</u> and is determined clockwise from geodetic north in opposition to the conventional geodesy reference clockwise from geodetic south. The length of the chord connecting P_1 and P_2 is found by the Pythagorean relation in the Cartesian coordinate system. $P_1(\phi_1, \lambda_1)$ and $P_2(\phi_2, \lambda_2)$ are converted to X, Y, Z coordinates by:

$$\begin{aligned} X &= a \cos \phi_p \cos \lambda \\ Y &= a \cos \phi_p \sin \lambda \\ Z &= \sqrt{1-e^2} \quad a \sin \phi_p \end{aligned} \qquad \text{(A-6)}$$

where ϕp is the parametric latitude. Parametric and geodetic latitude differ as follows: the parametric latitude is the angle at the center of the ellipsoid between the radius vector to the point of interest and the equatorial plane. The geodetic latitude is the angle between the normal to the surface at the point and the equitorial plane. The two latitudes are related by:

$$\tan \phi_p = \frac{b}{a} \tan \phi \qquad \text{(A-7)}$$

The points P_1 and P_2 are now in defined by $P_1(X_1,Y_1,Z_1)$ and $P_2(X_2,Y_2,Z_2)$. The length of the chord between the two points is:

$$D = ((\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2)^{1/2} \qquad \text{(A-8)}$$

where

$$\Delta X = X_2 - X_1$$
$$\Delta Y = Y_2 - Y_1 \qquad \text{(A-9)}$$
$$\Delta Z = Z_2 - Z_1$$

The length D is the geodesic approximation GLC of section 3.3.1.3.

The radius of curvature of the geodesic is used to define a sphere that approximates the spheroid at P_1 and/or P_2. The radius of curvature is found at one end of the geodesic for approximation GL1 of section 3.3.1.2 and averaged for both ends of the geodesic for approximation GL2 of section 3.3.1.1.

The radius of curvature of the geodesic is found at some point on the geodesic by the calculus (reference 7). The curvature depends on the azimuth of the geodesic at the point and is found trigonometrically by combining the radius of curvature of the meridian at the point of interest and the radius of curvature of prime vertical at the point. The radius of curvature of the meridian, M, depends on the geodetic latitude as:

$$M = \frac{a(1-e\ )^2}{(1-e^2\sin^2\phi)^{1/2}} \qquad \text{(A-10)}$$

The radius of curvature of the prime vertical, N, is also a function of geodetic latitude:

$$N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}} \qquad \text{(A-11)}$$

The radius of curvature of the geodesic at any point on the geodesic is found by:

$$R_\alpha = \frac{NM}{N\cos^2\alpha + M\sin^2\alpha} \qquad \text{(A-12)}$$

where α is the azimuth of the geodesic at the point of interest. For GL1, R_α is found using the forward azimuth at P_1. For GL2, R_α is found using the forward azimuth at P_1 and the back azimuth at P_2 and arithmetically averaging the two.

The radius of curvature of the geodesic (for GL1 or GL2) is now used to define a sphere of radius R equal to the radius of curvature. The length of the chord, D, between the two points P_1 and P_2 and the radius, R, are used to approximate the true length of the geodesic on the reference ellipsoid. From figure A-1:

$$S = 2R \sin^{-1}\left(\frac{D}{2R}\right) \tag{A-13}$$

where the inverse sine is calculated in radians.

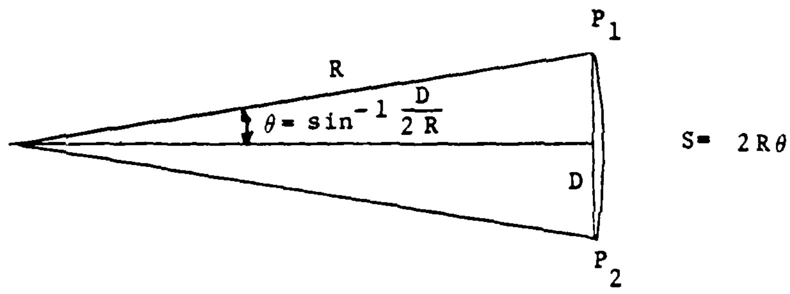

FIGURE A-1

A.2.2 Azimuth of Geodesics

The vector from P_1 to P_2 is:

$$\vec{D} = \Delta X\hat{i} + \Delta Y\hat{j} + \Delta Z\hat{k} \tag{A-14}$$

where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the x, y, and z directions, respectively. To find the azimuth, β , of the geodesic at P_1, it is desired to find the angle between a unit vector tangent to the surface at P_1 in the direction of geodetic north and a vector defined by projecting $\vec{D}$ onto a plane tangent to the spheroid at P_1. Define the following unit vectors at P_1:

$$\hat{n}_s = \frac{X_1\hat{i} + Y_1\hat{j} + \left(\frac{Z_1}{1-e^2}\right)\hat{k}}{\left(X_1^2 + Y_1^2 + \left(\frac{Z_1}{1-e^2}\right)^2\right)^{1/2}} \tag{A-15}$$

which is normal to spheroid at X_1, Y_1, Z_1.

$$\hat{\lambda} = \frac{-Y_1\hat{i} + X_1\hat{j}}{(X_1^2 + Y_1^2)^{1/2}} \tag{A-16}$$

which is in the direction of increasing longitude.

$$\hat{\phi} = \hat{n}_s \times \hat{\lambda} \tag{A-17}$$

which is in the direction of increasing latitude.

The vector $\vec{D}$ is projected onto the plane defined by $\hat{\lambda}$ and $\hat{\phi}$. The projected vector, $\vec{D}_p$, has components in the $\hat{\lambda}$ and $\hat{\phi}$ direction as follows:

$$\vec{D}_{\hat{\lambda}} = (\vec{D} \cdot \hat{\lambda})\,\hat{\lambda} \tag{A-18}$$

$$\vec{D}_{\hat{\phi}} = (\vec{D} \cdot \hat{\phi})\,\hat{\phi} \tag{A-19}$$

and

$$\vec{D}_p = \vec{D}_{\hat{\lambda}} + \vec{D}_{\hat{\phi}} \tag{A-20}$$

The angle, α, between $\vec{D}_p$ and $\hat{\phi}$ is used to find the desired azimuth. α is found by,

$$\alpha = \cos^{-1}\left(\frac{\vec{D}_p \cdot \hat{\phi}}{|\vec{D}_p|}\right) \tag{A-21}$$

and the forward azimuth is

$$\beta_F = \begin{cases} 360 - \alpha & \lambda_2 < \lambda_1 \\ \alpha & \lambda_2 \geq \lambda_1 \end{cases} \tag{A-22}$$

and the back azimuth (calculated similarly as β_F) at P_2 is

$$\beta_B = \begin{cases} 360 - \alpha & \lambda_2 < \lambda_1 \\ \alpha & \lambda_2 \leq \lambda_1 \end{cases} \tag{A-23}$$

APPENDIX B

MATHEMATICS OF PLANNING

The least-squares principle is employed to define the mathematical planning model from which expected results can be obtained. The principle is discussed extensively elsewhere (references 2, 9, 13, 14) and leads to the following matrix equation:

$$\underline{X} = (\underline{A}^T\, \underline{W}\, \underline{A})^{-1}\, \underline{A}^T\, \underline{W}\, \underline{L} \qquad \text{(B-1)}$$

where

$$\underline{X} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \qquad \underline{W} = \sigma_0^2 \quad \underline{\Sigma}_h^{-1} = \begin{bmatrix} \dfrac{\sigma_0^2}{\sigma_1^2} & & \overline{0} \\ & \ddots & \\ \overline{0} & & \dfrac{\sigma_0^2}{\sigma_n^2} \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} \dfrac{\partial m_1}{\partial x} & \dfrac{\partial m_1}{\partial y} \\ \vdots & \vdots \\ \dfrac{\partial m_n}{\partial x} & \dfrac{\partial m_n}{\partial y} \end{bmatrix} \qquad \underline{L} = \begin{bmatrix} \Delta m_1 \\ \Delta m_2 \\ \vdots \\ \Delta m_n \end{bmatrix}$$

Δx = x component (east) of vector from AP to AT
Δy = y component (north) of vector from AP to AT
Δm_i = $\alpha_{oi} - \alpha_{ci}$
σ_0^2 = arbitrary reference variance
m_i = i^{th} measurement (function of x and y coordinates)
α_{oi} = i^{th} observed measurement corrected for known systematic errors
α_{ci} = i^{th} computed angle at desired position
$\underline{\Sigma}_h$ = coveriance matrix of observations (diagonal)

This matrix equation is equivalent to a simpler set of equations for planning. The following information is needed for each line of position and is providied by equations in section 4.2:

a. The positive gradient magnitudes, G_i
b. The positive gradient directions, γ_i
c. The assumed measurement random errors, σ_i, for each instrument type.

The simpler set of equations is used to define position error measures needed (1) for fix geometry selection criteria, (2) to determine expected results, and (3) for standard setting.

The simpler set of equations is derived as follows:

The origin of the coordinate system is the desired position of the aid; the +y-axis=North, and +x-axis=East. Passing through the origin with positive gradient directions, γ_i, are the available lines of position. The angles γ_i are w.r.t. geodetic north. Select any *n* element subset ($n > 1$) of lines of position from those available (of course the γ_i must be different for n=2).

The matrix $\underline{A}$ is equivalent (figure B-1) to:

$$\underline{A} = \begin{bmatrix} \frac{\sin \gamma_i}{G_i} & \frac{\cos \gamma_i}{G_i} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \frac{\sin \gamma_n}{G_n} & \frac{\cos \gamma_n}{G_n} \end{bmatrix} \tag{B-2}$$

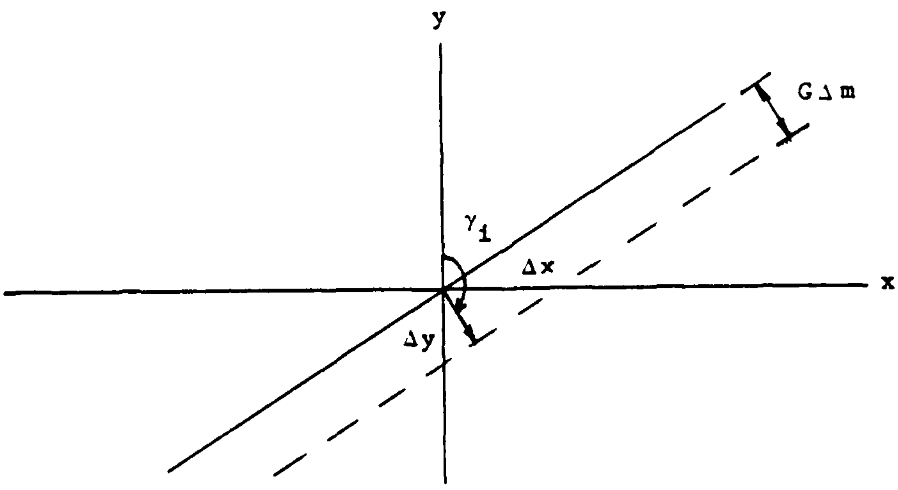

FIGURE B-1. $\underline{A}$ ELEMENT CONVERSION

The least squares equation now can be expressed as: (derivation for two measurement case for clarity).

$$\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} = \begin{bmatrix} a_{11}^2 w_{11} + a_{21}^2 w_{22} & a_{11}a_{12}w_{11} + a_{21}a_{22}w_{22} \\ a_{12}a_{11}w_{11} + a_{22}a_{21}w_{22} & a_{12}^2 w_{11} + a_{22}^2 w_{22}\end{bmatrix}^{-1} \begin{bmatrix} a_{11}w_{11}\Delta m_1 + a_{21}w_{22}\Delta m_2 \\ a_{12}w_{11}\Delta m_1 + a_{22}w_{22}\Delta m_2\end{bmatrix}$$

now let

$$A = \sum_{i=1}^{2} a_{i1}^2 w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}^2}\right) \qquad E = \sum_{i=1}^{2} a_{i2}^2 w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}^2}\right)$$

$$B = \sum_{i=1}^{2} \Delta m_i a_{i1} w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}}\right) \qquad F = \sum_{i=1}^{2} \Delta m_i^2 w_{ii} \quad \left(\text{min}^2\right)$$

$$C = \sum_{i=1}^{2} a_{i1} a_{i2} w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}^2}\right) \qquad G = \sum_{i=1}^{2} (1\ \text{min})\ a_{i2} w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}}\right)$$

$$D = \sum_{i=1}^{2} \Delta m_i a_{i2} w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}}\right) \qquad H = \sum_{i=1}^{2} (1\ \text{min})\ a_{i1} w_{ii} \quad \left(\frac{\text{min}^2}{\text{m}}\right) \tag{B-3}$$

with

$$w_{ii} = \frac{\sigma_0^2}{\sigma_i^2} \quad \text{and } \sigma_0^2 \text{ arbitrarily in min}^2$$

Now

$$\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} = \frac{1}{AE-C^2}\begin{bmatrix} EB-CD \\ -CB+AD\end{bmatrix} \tag{B-4}$$

or

$$\Delta x = \left[\frac{BE-CD}{AE-C^2}\right] \qquad \Delta y = \left[\frac{AD-BC}{AE-C^2}\right] \tag{B-5}$$

The AP-to-AT vector is defined by Δx and Δy.

$$\vec{V} = \Delta x\,\hat{i} + \Delta y\,\hat{j} \qquad \text{(B-6)}$$

where $\hat{i}$ and $\hat{j}$ are unit vectors in the position x and y directions, respectively.

The direction β_{AT}, w.r.t north, of $\vec{V}$ is,

$$\beta_{AT} = \begin{array}{c|c|c} & \Delta x \geq 0 & \Delta x < 0 \\ \hline \Delta y \geq 0 & \tan^{-1}\frac{\Delta x}{\Delta y} & 360^{o} + \tan^{-1}\frac{\Delta x}{\Delta y} \\ \hline \Delta y < 0 & 180^{o} + \tan^{-1}\frac{\Delta x}{\Delta y} & 180^{o} + \tan^{-1}\frac{\Delta x}{\Delta y} \end{array} \quad 0^{o} \leq \beta_{AT} < 360^{o} \qquad \text{(B-6)}$$

In planning, the $\underset{\sim}{L}$ matrix is initially defined with all zero elements because all lines of position are defined to pass through the desired position. Thus both Δx and Δy are zero. The usefulness of this mathematical relation in planning is to answer the following question:

> "If the measurements are in error, how serious is the resulting position error?"

This question can be answered (demonstrated here for sextant case) by entering a known systematic error into the $\underset{\sim}{L}$ matrix; for example, all elements of $\underset{\sim}{L}$ are 1 minute. The summation notation for this is provided by G and H of equations (B-3). The resulting vector is called the systematic error tendency (SET) and is defined by:

$$\Delta x_{set} = \frac{HE-CG}{AE-C^2}$$
$$\Delta y_{set} = \frac{AG-HC}{AE-C^2} \qquad \text{(B-8)}$$

The obvious use of this is to examine all possible fix geometries to determine which ones are least susceptible to inaccuracy.

The component of $\vec{V}$ in a predesignated direction is discussed in appendix D as a possible measure for use in setting standards.

The derivation of selection and prediction equations now turns to the stochastic nature of the observations. The random measurement error associated with each line of position and the arbitrary reference variance have been assumed for planning purposes. These assumptions allow determination of the confidence in the determined position. The confidence is described in terms of a two-dimensional probability distribution centered on the desired position (no

systematic error involved). Various descripters of this distribution are defined as position error measures. The derivation is as follows.

The matrix of assumed measurement variances is:

$$\underline{\Sigma}_h = \begin{bmatrix} \sigma_1^2 & & & & \overline{0} \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ \overline{0} & & & & \sigma_n^2 \end{bmatrix} \qquad \text{(B-9)}$$

Define

$$\underline{W} = \sigma_0^2 \; \underline{\Sigma}_h^{-1}$$

By a Jacobian transformation (reference 13), the matrix of coordinate variances is

$$\underline{\Sigma}_x = \sigma_0^2 \; (\underline{A}^T \; \underline{W} \; \underline{A})^{-1} = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \qquad \text{(B-10)}$$

For two measurements (for clarity), $\underline{\Sigma}_x$, is reduced to

$$\underline{\Sigma}_x = \begin{bmatrix} a_{11}^2 w_{11} + a_{21}^2 w_{22} & a_{11} a_{12} w_{11} + a_{21} a_{22} w_{22} \\ a_{12} a_{11} w_{11} + a_{22} a_{21} w_{22} & a_{12}^2 w_{11} + a_{22}^2 w_{22} \end{bmatrix}^{-1} \sigma_0^2 \qquad \text{(B-11)}$$

$$\underline{\Sigma}_x = \frac{\sigma_0^2}{AE - C^2} \begin{bmatrix} E & -C \\ -C & A \end{bmatrix}$$

Normally, a correlation exists between coordinates and $C \neq 0$. To remove this correlation (reference 13) the axes are rotated clockwise a.1 angle β_{CE} according to

$$\beta_{CE} = 1/2 \tan^{-1}\left(\frac{-2C}{A-E}\right) \qquad \text{(B-12)}$$

With the following conditions, β_{CE} represents the orientation of the major semi-axis w.r.t. north:

$$\text{if } A=E \text{ and } C \begin{cases} < 0 \\ = 0 \\ > 0 \end{cases} \quad \text{then } \beta_{CE} = \begin{cases} 45^{\circ} \\ 0^{\circ} \\ 135^{\circ} \end{cases}$$

if $A<E$ then the minor semi-axis has been found w.r.t. north and 90° must be added to find the orientation of the major semi-axis. Finally,

if $\beta_{CE} < 0$ add 180°

The new uncorrelated (u,v) coordinate system allows the covariance matrix to be defined as

$$\underline{\Sigma}_u = \sigma_0^2 \begin{bmatrix} A_1^2 & 0 \\ 0 & B_1^2 \end{bmatrix} = \begin{bmatrix} \sigma_{maj}^2 & 0 \\ 0 & \sigma_{min}^2 \end{bmatrix} \qquad \text{(B-13)}$$

with

$$\sigma_{maj}^2 = \sigma_0^2 A_1^2 = \frac{2\sigma_0^2}{A+E-((A-E)^2+4C^2)^{1/2}}$$

$$\sigma_{min}^2 = \sigma_0^2 B_1^2 = \frac{2\sigma_0^2}{A+E+((A-E)^2+4C^2)^{1/2}} \qquad \text{(B-14)}$$

A_1 and B_1 are called the geometry factors.

The confidence ellipse is defined on the axi[illegible] uncorrelated coordinates (reference 12) by

$$\left(\frac{u-\mu_u}{\sigma_{maj}}\right)^2 + \left(\frac{v-\mu_v}{\sigma_{min}}\right)^2 = \chi^2_{2,\alpha} \qquad \text{(B-15)}$$

where α is the confidence level desired and μ_u and μ_v are the Cartesian coordinates of the AT. In the planning case, μ_u and μ_v, are zero unless systematic error is simulated.

The expected position error is given in terms of the bivariate normal probability distribution defined above. The major and minor semi-axes of a desired confidence ellipse are determined by α. The semi-axes are multiples of σ_{maj} and σ_{min} where the multiple, M_σ, is determined by

$$M_\sigma = \sqrt{\chi^2_{2,\alpha}} \tag{B-16}$$

The semi-axes are

$$\begin{aligned} A_\sigma &= M_\sigma \; \sigma_{maj} \\ B_\sigma &= M_\sigma \; \sigma_{min} \end{aligned} \tag{B-17}$$

Table B-3 provides a selected set of multiples and the corresponding confidence that it can be expected that the true position will lie within the specified ellipse centered on the AT (reference 17).

TABLE B-3

MULTIPLE	PROBABILITY LEVEL
$\sqrt{4.605} = 2.15$	90%
$\sqrt{5.991} = 2.45$	95%
$\sqrt{9.210} = 3.03$	99%

The area of the one standard deviation ellipse is determined as follows (det represents taking the determinant of the matrix):

$$\text{Area}_\sigma = \pi\,(\det \underline{\Sigma}_u)^{1/2} = \pi\,(\det (\sigma_0^2\, \underline{P}^{-1}(\underline{A}^T \underline{W}\, \underline{A})^{-1}\underline{P}))^{1/2} \tag{B-18}$$

where $\underline{P}$ rotates $\underline{\Sigma}_u$ to uncorrelate coordinates. Continuing,

$$\begin{aligned} \text{Area}_\sigma &= \pi\,(\sigma_0^4\, |\underline{P}^{-1}(\underline{A}^T \underline{W}\, \underline{A})^{-1}\underline{P}|)^{1/2} = \pi\,\sigma_0^2 \left[\, |\underline{P}^{-1}| \;\; |(\underline{A}^T \underline{W}\, \underline{A})^{-1}| \;\; |\underline{P}| \,\right]^{1/2} \\ &= \frac{\pi\,\sigma_0^2}{|\underline{A}^T \underline{W}\, \underline{A}|^{1/2}} = \frac{\pi\,\sigma_0^2}{(AE-C^2)^{1/2}} \end{aligned} \tag{B-20}$$

The area of other confidence ellipses are determined by:

$$\text{Area}_\sigma = \frac{\pi\, M_\sigma^2\, \sigma_0^2}{(AE-C^2)^{1/2}} \tag{B-21}$$

where M_σ is determined at some confidence level, α .

In some situations the confidence ellipse semi-diameter in a specified direction is important.

Three previously defined quantities are needed to calculate the semi-diameter of a particular confidence ellipse in a direction, δ , clockwise from geodetic north. They are σ^2_{maj}, σ^2_{min}, and the ellipse orientation angle, β_{CE}. The major semi-diameter oriented in a direction δ is

$$A_{\sigma\delta} = \frac{M_\sigma}{\left(\dfrac{\sin^2(\delta - \beta_{CE})}{\sigma^2_{min}} + \dfrac{\cos^2(\delta - \beta_{CE})}{\sigma^2_{maj}}\right)^{1/2}} \tag{B-22}$$

Another position error measure useful in planning is the probability mass (of the p.d.f. centered about the AT) contained within a designated circular region of radius R (centered on the AP), called P-in-R. Calculation of this measure is outlined in appendix G of reference 2. P-in-R is a function combining both the accuracy and precision in positioning. Further discussion of P-in-R is contained in appendix D herein which discusses standards.

The final expected position error measure is the 2-drms value. It is found by

$$\text{2-drms} = 2\sqrt{\sigma^2_{maj} + \sigma^2_{min}} \tag{B-23}$$

A circle with a radius of 2-drms centered on the center of the ellipse encloses at least 95% of the probability mass.

APPENDIX C

MATHEMATICS OF POSITIONING

C.1 COORDINATE SYSTEM FOR ERROR ANALYSIS AND NOTATION

The origin of the coordinate system is the assumed position. The $+x$ direction is to the east and the $+y$ direction is to the north. Each fix geometry of n measurements consists of n:

a. Lines of position with their gradient magnitude and direction, G_i and γ_i, and relative weights, w_{ii}.
b. Precomputed measurements, α_{ci}.
c. Observed measurements (corrected for systematic errors), α_{oi}.

C.2 DETERMINATION OF THE ANGLE TAKERS POSITION (AT)

The equations to calculate the AP-to-AT vector in planning are used iteratively in calculating the AP-to-AT vector when positioning. Each iteration differs only by the choice of the assumed position. The desired position is used for the first iteration and successively determined AT's are used as the AP for following iterations. This process accounts for error due to the linearization of the system. The final values of Δx and Δy are the sums of all incremented changes in x and y. The sums are used to convert back to latitude and longitude with equations (C-4).

From appendix B the components of the AP-to-AT vector are:

$$\Delta x = \frac{BE-CD}{AE-C^2} \quad \text{(summed over all iterations)}$$

$$\Delta y = \frac{AD-BC}{AE-C^2} \quad \text{(summed over all iterations)} \tag{C-1}$$

The magnitude of the AP-to-AT vector, $\vec{V}$, is:

$$|\vec{V}| = (\Delta x)^2 + (\Delta y)^2 \tag{C-2}$$

The direction, β_{AT}, of $\vec{V}$, w.r.t. geodetic north, is:

$$\beta_{AT} = \begin{array}{c|c|c|} & \Delta x \geq 0 & \Delta x < 0 \\ \hline \Delta y \geq 0 & \tan^{-1}\frac{\Delta x}{\Delta y} & 360^\circ + \tan^{-1}\frac{\Delta x}{\Delta y} \\ \hline \Delta y < 0 & 180^\circ + \tan^{-1}\frac{\Delta x}{\Delta y} & 180^\circ + \tan^{-1}\frac{\Delta x}{\Delta y} \\ \hline \end{array} \quad 0^\circ \leq \beta_{AT} < 360^\circ \tag{C-3}$$

The conversion of the Δx and Δy (which are in meters) to $\Delta\lambda$ and $\Delta\phi$ is accomplished through (reference17):

$$\Delta\phi = (\text{Diff. per. sec.})\ \Delta y$$

(converts Δy in meters to $\Delta\phi$ in seconds)

$$\Delta\lambda = H\ \Delta x \qquad \text{(C-4)}$$

(converts Δx in meters to $\Delta\lambda$ in seconds)

where from USC&GS S.P#241 (other references exist that provide these equations in slightly different forms),

$$\text{Diff. per. sec.} = 1/(111132.09 - 566.05\cos 2\phi_{AP} + 1.2\cos 4\phi_{AP}) \qquad \text{(C-5)}$$

$$H = -\text{Diff. per. sec.}/\cos\phi_{AP} \qquad \text{(C-6)}$$

The geographic latitude and longitude of the AT are found by:

$$\lambda_{AT} = \lambda_{AP} + \Delta\lambda$$
$$\phi_{AT} = \phi_{AP} + \Delta\phi \qquad \text{(C-7)}$$

If the angle takers are displaced from the chain stopper, the vectors $\vec{V}$ and $\vec{D}_s$ (appendix C.3) should be added before finding geographic coordinates.

C.3 SYSTEMATIC ERRORS

C.3.1 Lack of Observer Coincidence

It is desirable for observers to stand at the same point when making measurements. Of course, this is difficult due to inter-visibility conditions and therefore a systematic error of observer lack-of-coincidence exists and should be compensated for before computations are performed on the measurement set. Reference 2 explains the significance of the lack of coincidence. It was found to be small in most cases but the equations to compensate for the error are presented here for completeness.

Define the displacement vector, $\vec{O}_i$, to each observer from some reference point in the region where they make measurements. The corrected observation is α_{oi}, which represents the measurement that would have been made if the observer were standing at the reference point. The corrected observation is found as follows. The gradient vector corresponding to each LOP is $\vec{G}_i$. The vectors $\vec{G}_i$ and $\vec{O}_i$ are used to find α_{oi} from the uncorrected measurement, α_i.

$$\alpha_{oi} = \alpha_i + \frac{\vec{O}_i}{|\vec{G}_i|} \cdot \frac{\vec{G}_i}{|\vec{G}_i|} = \alpha_i + \frac{|\vec{O}_i|}{|\vec{G}_i|}\cos(\beta_h + \beta_{oi} - \gamma_i)$$

where the direction of $\vec{O}_i$ is determined by the sum of the heading direction, β_h, and the direction $\vec{O}_i$ makes the clockwise from β_h, which is called β_{oi}.

C.3.2 Angle Taker to Sinker Drop Point Vector

Observers are not able to stand near the sinker drop point during positioning operations. The displacement vector, $\vec{D}_s$, from the observation point to the sinker drop point may be compensated for after preliminary determination of the position of the angle takers (AT). The vector from the desired position, AP, to the calculated position of the observers (AT) is $\vec{V}$. Let β_h represent the true heading of the positioning platform (clockwise from north) and let β_s represent the direction of $\vec{D}_s$ clockwise from β_h. The AP-to-MPP vector, $\vec{V}_c$, is computed by

$$\vec{V}_c = \vec{V} + \vec{D}_s \qquad \text{(C-8)}$$

The direction of $\vec{D}_s$ is the sum of β_h and β_s. The direction of $\vec{V}$ was discussed in appendix C.2.

The angle taker to sinker drop point vector can also be compensated for by placing a scaled model of the positioning unit on the gradient diagram.

C.4 DETERMINATION OF PRECISION IN POSITIONING

The residuals of a fix can be used to determine the precision associated with the determined position. A residual is the difference between an observed measurement and an adjusted measurement. The least squares procedure minimizes the sum of the squares of weighted residuals. In matrix notation, the residuals are (using the matrix notation of appendix B).

$$\underline{R} = \underline{AX} - \underline{L} \qquad \text{(C-9)}$$

The sum of the weighted squared residuals is:

$$\begin{aligned}\underline{R}^T\underline{WR} &= (\underline{AX} - \underline{L})^T\, \underline{W}(\underline{AX} - \underline{L}) \\ &= \underline{X}^T\underline{A}^T\underline{WAX} - \underline{X}^T\underline{A}^T\underline{WL} - \underline{L}^T\underline{WAX} + \underline{L}^T\underline{WL}\end{aligned}$$

When $\underline{R}^T\underline{WR}$ is minimized (reference 13,31):

$$\begin{aligned}&\underline{X} = (\underline{A}^T\underline{WA})^{-1}\underline{A}^T\underline{WL} \\ &\underline{X}^T\underline{A}^T\underline{WAX} = \underline{X}^T\underline{A}^T\underline{WL} \\ &\underline{R}^T\underline{WR} = \underline{L}^T\underline{WL} - \underline{L}^T\underline{WAX}\end{aligned} \qquad \text{(C-10)}$$

An unbiased, most likely a posteriori estimate, s^2, of the reference variance, σ_0^2, is provided by (references 13,31):

$$s^2 = \frac{R^T W R}{n-2} = \frac{L^T W L - L^T W A X}{n-2} \tag{C-11}$$

where n-2 is the number of degrees of freedom of the estimate. Two degrees of freedom were lost to determining the most probable position.

The estimator, s^2, can be reduced to summation notation (see appendix B) as follows (for 2 measurement case; higher numbers follow readily),

$$(n-2)s^2 = [\Delta m_1 \quad \Delta m_2] \begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix} \begin{bmatrix} \Delta m_1 \\ \Delta m_2 \end{bmatrix} - [\Delta m_1 \quad \Delta m_2] \begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$(n-2)\ s^2 = \Delta m_1{}^2 w_{11} + \Delta m_2{}^2 w_{22} - \Delta m_1 w_{11} a_{11}\,\Delta x - \Delta m_2 w_{22} a_{21}\,\Delta x - \Delta m_1 w_{11} a_{12}\,\Delta y - \Delta m_2 w_{22} a_{22}\,\Delta y \tag{C-12}$$

and finally

$$s^2 = (F - B\ \Delta x - D\ \Delta y)/n-2 \tag{C-13}$$
(units of reference variance)

The variate $(n-2)\ s^2/\sigma_0^2$ is χ^2_{n-2} distributed and is used to calculate the confidence ellipse dimensions.

From equation (B-15), the confidence ellipse on the axis of uncorrelated coordinates is defined as,

$$\left[\frac{u - \mu_u}{\sigma_{maj}}\right]^2 + \left[\frac{v - \mu_v}{\sigma_{min}}\right]^2 = \chi^2_{2,\alpha} \tag{C-14}$$

The reference variance, σ_0^2, is related to σ_{maj}^2 and σ_{min}^2 by equations (B-14). Using equations (B-13) in (B-14),

$$\left[\frac{u - \mu_u}{A_1 \sigma_0}\right]^2 + \left[\frac{v - \mu_v}{B_1 \sigma_0}\right]^2 = \chi^2_{2,\alpha} \tag{C-15}$$

By dividing both sides by $(n-2)s^2/\sigma_0^2$

$$\frac{\left(\frac{u - \mu_u}{A_1 \ 0}\right)^2}{\frac{(n-2)\ s^2}{\sigma_0^2}} + \frac{\left(\frac{v - \mu_v}{B_1 \ 0}\right)^2}{\frac{(n-2)\ s^2}{\sigma_0^2}} = \frac{\chi^2_{2,\alpha}}{\chi^2_{n-2,\alpha}}$$

an F-distributed variate arises using Generalized-T^2 statistics (references 12 and 15).

$$\left(\frac{u - \mu_u}{A_1 s}\right)^2 + \left(\frac{v - \mu_v}{B_1 s}\right)^2 = 2\ F_{2,n-2,\alpha} \tag{C-16}$$

$$\left(\frac{u - \mu_u}{s_{maj}}\right)^2 + \left(\frac{v - \mu_v}{s_{min}}\right)^2 = M_s^2 \tag{C-17}$$

where $M_s = \sqrt{2F_{2,n-2,\alpha}}$

M_s is found for different values of n and α. Table C-1 provides a set of selected M_s values for various n and α (reference 17). The major and minor semi-axes of the confidence ellipse are:

$$\begin{aligned} A_s &= M_s\ s_{maj} = M_s\ A_1\ s \\ B_s &= M_s\ s_{maj} = M_s\ B_1\ s \end{aligned} \tag{C-18}$$

TABLE C-1

CONFIDENCE ELLIPSE MULTIPLIERS

Confidence	n = 2	3	4	5
90%	∞	9.94	4.24	3.30
95%	∞	19.97	6.16	4.37
99%	∞	100.00	14.07	7.85

Equation (C-16) demonstrates that only the relative weights of the measurements are important to confidence ellipse determination. The relative weights are used in calculation of A_1, B_1, and s. If only one measurement type is used, all weights are equal and the reference variance is that of the measurement type used.

The orientation of the major semi-axis w.r.t. north is provided by equation (B-12) as i'. was for planning.

The semi-diameter, $A_{s\delta}$, of the confidence ellipse of confidence level in some direction, δ, clockwise from geodetic north, can be determined by:

$$A_{s\delta} = \frac{M_s\ s}{\left(\dfrac{\sin^2(\delta - \beta_{CE})}{A_1^2} + \dfrac{\cos^2(\delta - \beta_{CE})}{B_1^2}\right)^{1/2}} \qquad \text{(C-19)}$$

The area of a confidence ellipse of confidence level α is,

$$\text{Area}_s = \pi M_s^2\ A_1 B_1 s^2 = \frac{\pi\ M_s^2\ s^2}{(AE-C^2)^{1/2}} \qquad \text{(C-20)}$$

where M_s is determined by α and the number of measurements.

A measure of position error which combines the major and minor semi-axis into one number is the circle of confidence, COC. The circle of confidence encloses at least $1-\alpha$ of the probability mass and is found by

$$COC = M_s\ s\sqrt{A_1^2 + B_1^2} \qquad \text{(C-21)}$$

The circle of confidence is centered on the most probable position.

C.5 REPLICATION - TIME AVERAGING LOPS

Suppose that the ability to replicate each of the n lines of position in a fix geometry m times exists (e.g., LORAN receivers do this). The effect of time averaging "n times m" measurements on fix precision is discussed in this appendix.

All position error measures of precision are functions of the a posteriori estimate of the reference variance, the confidence level multiple, M_s (appendix C) and the geometry factors, A_1 and B_1. If each of the n lines is measured with equal precision, the geometry factors are not dependent on the number of replications. The main influence on precision is through M_s and s^2.

For n lines of position:

$$s^2 = \frac{\underline{R}^T \underline{W} \underline{R}}{n-2} \qquad \text{(C-22)}$$

Let each residual consist of a systematic component, r_s, and a random component, r_r. The residual matrix is now written as:

$$\underline{R} = \underline{R_s} + \underline{R_r}$$

The estimate is:

$$s^2 = \frac{(\underline{R}_s + \underline{R}_r)^T \underline{W} (\underline{R}_s + \underline{R}_r)}{n-2} \qquad \text{(C-23)}$$

Replicate each line of position m times randomly and average the residuals. The random components will average close to zero if m is large (>100) and only the systematic components remain. Thus,

$$s^2 = \frac{\underline{R}_s^T \underline{W}\, \underline{R}_s}{n-2} \qquad \text{(C-24)}$$

If no systematic error exists, the estimate of the reference measurement variance would be near zero.

The confidence level multiplier M_s is given by equation (C-16),

$$M_s = \sqrt{2\, F_{2,n-2,\alpha}} \qquad \text{(C-25)}$$

As the number of measurements gets large (with $\alpha = .1$), $F_{2,n-2,.1}$ approaches 2.3 and M_s approaches 2.15. This is(not coincidentally) the same as $\chi^2_{2,.1}$.

The geometry factor A_1 is not dependent on the number of replications made of each line of position as it is computed using the orientations and gradients of each of the n lines of position.

The effect of m replications is displayed by the equation (C-18) for the major semi-axis.

$$A_s = M_s A_1 s = 2.15\, A_1 \sqrt{\frac{\underline{R}_s^T \underline{W}\, \underline{R}_s}{n-2}} \qquad \text{(C-26)}$$

The result of this derivation indicates that little is to be gained by time averaging of lines of position if systematic error exists. There is an upper limit on the precision with which a position can be determined. The results of this section dispel the common belief by many mariners that time replication always provides a very accurate fix with high precision.

APPENDIX D

STANDARDS-POSITION ERROR MEASURES

The purpose of this appendix is to explore various alternatives for a position error measure, S, when considering standards for aid positioning. A standard is defined as an authoritative measure, S_m, for comparison with S to determine if S is acceptable, which indicates that the positioning evolution has been acceptable. S_m may depend on the specific environment at the time of positioning, the availability of signals, the quality of the signals, the importance or criticality of the aid, and the difficulty in achieving S_m (i.e., limit on number of attempts at reaching S_m) at the time of positioning. More than one S_m value may be applicable to any one positioning evolution. The specific combination depends on the requirements placed upon the aid at the time. Successful aid positioning is determined by both accuracy and precision; this means the applicable set of S_m values must at least contain tests for these two qualities.

Frequency histograms of position error measures found through extensive research of historical data received from the REDWOOD (WLM-685) are provided at the end of this appendix. The data for each position error measure are separated only by chart scale. All fixes are 3-line fixes using Third Order or better reference objects.

D.1 A PORTERIORI ESTIMATE OF REFERENCE VARIANCE

The reference variance estimate, s^2, of equation (C-11) can be used as a measure of the precision of a fix (reference 13). The estimate is unbiased and can be tested against some a priori value, σ_0^2, using the χ^2 test. With *n* measurements,

$$\chi^2_{n-2} = \frac{(n-2)\ s^2}{\sigma_0^2} \tag{D-1}$$

has a χ^2 distribution with n-2 degrees of freedom. There are two possible hypothesis tests. First,

$$H_0 : \quad \sigma^2 = \sigma_0^2 \quad \text{versus} \quad H_1 : \quad \sigma^2 \neq \sigma_0^2 \tag{D-2}$$

and reject H_0 when

$$\chi^2_{n-2} < \chi^2_{n-2, 1-\alpha/2} \quad \text{or} \quad \chi^2_{n-2} > \chi^2_{n-2, \alpha/2} \tag{D-3}$$

Second,

$$H_0 : \sigma^2 = \sigma_0^2 \text{ versus } H_1 : \sigma^2 > \sigma_0^2 \tag{D-4}$$

and reject H_0 when $\chi^2_{n-2} > \chi^2_{n-2,\alpha}$. The second is more applicable to the OPM.

The standard is selected at some α such that

$$\frac{(n-2)\, s^2}{\sigma_0^2} > S_m \text{ where } S_m = \chi^2_{n-2,\,\alpha} \tag{D-5}$$

indicates an adequate positioning evolution. σ_0^2 need not be constant or even representative of measurement random error. It can represent some critical value based on other considerations beyond which rejection occurs. It is most advisable, however, to establish σ_0^2 through analysis of historical data.

Figure D-1 is a cumulative frequency histogram of s for fixes performed by the crew of the REDWOOD (WLM-685). The data has been separated into two groups by chart scale. The figures indicate that positioning efforts on aids found on charts of scale 1:20,000 or less result in s values of on the average 7.2 minutes. Positioning of aids located on larger scale charts leads to an average s of 4.4 minutes. Remember s is an unbiased estimates of σ_0.

D.2 A POSTERIORI ESTIMATE OF LOP VARIANCE

The distance residuals are defined as the distances in meters between the AT and the LOPs along a line perpendicular to the LOPs. In terms of the measurement residuals, the matrix of distance residuals is given by

$$\underline{D} = \underline{G}\,\underline{R} = \underline{G}\,(\underline{A}\,\underline{X} - \underline{L}) \tag{D-6}$$

where $\underline{G}$ is an nxn diagonal matrix of LOP gradient magnitudes.

The distance residuals are weighted by the usual weighting matrix defined in equation (B-1), squared and summed.

The sum is differentiated and set equal to zero which leads to

$$\underline{X} = (\underline{A}^T\underline{G}^T\underline{WGA})^{-1}\underline{A}^T\underline{G}^T\underline{WGL} \tag{D-7}$$

This result is identical with that presented in appendix B with $\underline{W}$ redefined as $\underline{G}^T\underline{W}\underline{G}$. In this case, the reference variance is in units of the LOP variance (meter^2). One way to calculate the LOP reference variance is

$$\sigma^2_{lop_0} = \frac{1}{n}\left(\sum_{i=1}^{n} G_i^2\ \sigma_i^2\right) = \overline{G^2\ \sigma^2} \tag{D-8}$$

where the σ_i^2 are the assumed measurement variances.

The calculation of the reference variance estimate can be altered to provide an LOP variance estimate. The estimate is found as follows:

$$\underline{D}^T\underline{W}\underline{D} = \underline{L}^T\underline{G}^T\underline{W}\underline{G}\underline{L} - \underline{L}^T\underline{G}^T\underline{W}\underline{G}\underline{A}\underline{X}$$

An unbiased most likely a posteriori estimate of the LOP variance, s^2_{lop}, is provided by

$$s^2_{lop} = \frac{\underline{D}^T\underline{W}\underline{D}}{n-2} = \frac{\underline{L}^T\underline{G}^T\underline{W}\underline{G}\underline{L} - \underline{L}^T\underline{G}^T\underline{W}\underline{G}\underline{A}\underline{X}}{n-2} \tag{D-9}$$

where n-2 is the number of degrees of freedom in the estimate. The possible hypothesis are similar to those in the previous section.

The confidence level, α, can be selected to account for the conditions at the location of the aid. The standard is selected at some α such that

$$\frac{(n-2)\ s^2_{lop}}{\sigma^2_{lop_0}} < S_m \text{ where } S_m = \chi^2_{n-2,\alpha} \tag{D-10}$$

indicates an acceptable positioning evolution.

Figure D-2 is a cumulative frequency histogram of s_{lop} for fixes performed by the crew of the REDWOOD. For small-scale charts, s_{lop} averages 6.8 meters and for large-scale charts, s_{lop} averages 9.9 meters. Remember, s_{lop} is an unbiased estimate of σ_{lop_0}.

D.3 CONFIDENCE ELLIPSE PARAMETERS

The dimensions and orientation of the confidence ellipse provide four position error measures, S, at any confidence level, α. They are:

a. The major semi-axis.
b. The area of the confidence ellipse.
c. Semi-diameter in specified direction δ.
d. The square root of the sum of the squares of the major and minor semi-axes. Table D-1 directs the reader to the equations needed to calculate the position error measures.

TABLE D-1

POSITION ERROR MEASURES FOR CONFIDENCE ELLIPSE

MEASURE	EQUATION
A_s	C-18
$A_{s\delta}$	C-19
$Area_s$	C-20
COC	C-21

Figures D-3, D-4, and D-5 are cumulative frequency histograms of confidence ellipse parameters for CGC REDWOOD fixes. Average results are:

SCALE	$A_s(0.90)$	$Area_s(0.90)$	COC (0.90)
$\leq$ 20	39 meters	4,200 m^2	43.3 meters
> 20	66 meters	12,400 m^2	75.8 meters

D.4 AP-to-MPP

The magnitude and direction of the AP-to-MPP vector, $\vec{V}_c$, is calculated using equations (C-1), (C-2), (C-3), and (C-8). Setting a standard on the magnitude of $\vec{V}_c$ is equivalent to establishing the level of acceptable accuracy; acceptable precision is not established using $\vec{V}_c$. In many situations the component of $\vec{V}_c$ in a specific direction may be important. For example, in marking a narrow channel, the component along the channel is of little importance relative to the transverse component. The component of $\vec{V}_c$ in any direction ψ, clockwise from geodetic north, is found by

$$\vec{V}_{c\psi} = (\vec{V}_c \cdot \hat{t})\hat{t} = |\vec{V}_c| \cos(\beta_{MPP} - \psi)\hat{t} \qquad \text{(D-11)}$$

where $\hat{t}$ is a unit vector at the AP in the ψ direction. The standard, S_m, is established such that,

$$|\vec{V}_c| < S_m \qquad \text{(D-12)}$$

or

$$|\vec{V}_{c\delta}| < S_m \qquad \text{(D-13)}$$

indicates an acceptable positioning evolution.

Figure D-6 is a cumulative frequency histogram of the magnitude of the AP-to-AT vector for REDWOOD fixes. The average vector magnitude for the small-scale chart case is 13.1 meters. The large-scale case gives a magnitude of 23.4 meters.

D.5 P-in-R

The probability mass (of the p.d.f. centered about the MPP) contained within a designated circular region of radius R (centered on the AP) is called P-in-R. Calculation of this success measure is outlined in appendix G of reference 2. P-in-R is a function combining both the precision and the accuracy in positioning. Calculation of P-in-R requires two-dimensional numerical integration (which is very time consuming); numerical error in calculating P-in-R is largely dependent upon:

a. The number of points at which the integrand is evaluated.
b. The limits of the integration.
c. The quadrature method employed.
d. The higher order derivatives of the integrand.

Table D-2 displays the relative error in calculating P-in-R using various orders or Gaussian Quadrature (reference 18) for an assortment of R/σ_{maj} and $\sigma_{maj}/\sigma_{min}$ ratios. The relative errors are presented as the percent difference between the calculated values and the true P-in-R for each case. The time required to calculate P-in-R is a function of the order of integration. From the table,

$$\text{Time (secs)} = 5\ (\#\ \text{pts})^{3/2}\ \text{(on HP-41C)} \tag{D-14}$$

where (# pts) is the order of Gaussian Quadrature.

The regions of the table where the relative error is less than 10% are boxed in to indicate the applicability of P-in-R as a measure of success in positioning. The calculations indicate that:

a. P-in-R cannot be calculated accurately with low-order (<8) quadrature methods unless the

$$\frac{\sigma_{maj}}{\sigma_{min}} < 3 \text{ and } \frac{R}{\sigma_{maj}} < 3 \tag{D-15}$$

b. If P-in-R is to be calculated for accurate immediate feedback when positioning (<1 min), then,

$$\frac{\sigma_{maj}}{\sigma_{min}} + \frac{R}{\sigma_{maj}} < 5 \tag{D-16}$$

c. Accurate feedback in less than 15 seconds is restricted to

$$\frac{\sigma_{maj}}{\sigma_{min}} + \frac{R}{\sigma_{maj}} < 3 \tag{D-17}$$

TABLE D-2

PERCENT ERRORS IN CALCULATING P-IN-R AND COMPUTATION TIME (HP-41C)

$\sigma_{maj}/\sigma_{min}$	R/σ_{min}	ORDER OF INTEGRATION (#PTS//TIME IN MINUTES) 2//0.25m	4//0.67m	6//1.25m	8//2.00m	16//5.3m
	1	0	0	0	0	0
	2	21	0	0	0	0
1	3	61	7	0	0	0
	4	90	28	3	0	0
	5	99	59	14	2	0
	6	100	82	32	8	0
	1	4	0	0	0	0
	2	62	11	0	0	0
2	3	96	54	15	3	0
	4	100	87	48	19	0
	5	100	92	77	44	1
	6	100	100	93	68	5
	1	25	1	0	0	0
	2	94	47	13	9	0
3	3	100	91	59	28	0
	4	100	100	91	65	4
	5	100	100	99	89	17
	6	100	100	100	97	36
	1	55	9	0	0	0
	2	100	76	38	14	0
4	3	100	98	87	60	0
	4	100	100	100	92	21
	5	100	100	100	99	48
	6	100	100	100	100	62
	1	80	23	3	0	0
	2	100	90	63	33	0
5	3	100	100	97	84	10
	4	100	100	100	99	46
	5	100	100	100	100	75
	6	100	100	100	100	91
	1	93	40	10	2	0
	2	100	95	80	53	10
6	3	100	100	99	100	31
	4	100	100	100	100	69
	5	100	100	100	100	91
	6	100	100	100	100	98

Time = 5 $(\#pts)^{3/2}$ seconds

d. P-in-R can be calculated accurately if time is not an important restriction (> 5m) for,

$$\frac{\sigma_{maj}}{\sigma_{min}} + \frac{R}{\sigma_{maj}} < 8 \qquad \text{(D-18)}$$

In calculation of table 3-5, the AP-to-AT vector magnitude was assumed to be zero. $\overline{V_C}$ does affect the relative error but does so in a positive way. In fact, if $\overline{V_C}$ exceeds R, the accuracy of the integration is significantly improved. The reason for this being that the region of integration is where the p.d.f. changes slowly with position.

Figures D-7, D-8, and D-9 are cumulative frequency histograms of the P-in-R values calculated for REDWOOD fixes. The average P-in-Rs are as follows:

SCALE	RADIUS IN METERS R=10	R=15	R=20
≤ 20	0.47	0.68	0.80
> 20	0.10	0.19	0.34

D.6 R-for-P

The radius of a circular region (centered on the AP) that contains at least the probability mass P (of the p.d.f. centered on the MPP) is called R-for-P.

R-for-P is defined as the sum,

$$\text{R-for-P} = A_S + |\vec{V_C}| \qquad \text{(D-20)}$$

which is just the sum of the major semi-axis of the confidence ellipse and the magnitude of the AP-to-MPP vector. The standard, S_m, is established at some level α such that

$$\text{R-for-P} < S_m \qquad \text{(D-21)}$$

indicates an acceptable positioning evolution or classifies the position determined.

Although this approximation is rather crude at some R, the P is in error by no more than 10 percent and any numerical methods for calculating *R-for-P more accurately are extremely time consuming.*

Figure D-10 is a cumulative frequency histogram of R-for-P (for α = 0.1) from data taken from the REDWOOD. For small-scale charts, the average R-for-P is 51.6 meters. Fixes associated with aids on large-scale charts average 89.2 meters.

D.7 DIFFERENCE BETWEEN OBSERVED MEASUREMENTS AND COMPUTED MEASUREMENTS

D.7.1 All Measurements - Statistical Test

In section 4.2, procedures for precomputing measurements (expected to be observed at the assigned position) were discussed. After numerous operations at a given station, the positioning team should develop high confidence in the expected measurements. In cases where high confidence exists, measurements can and should be compared to those that are expected. The expected measurements have no estimated parameters, which causes the statistical comparison to be one level less complicated than the consistency checks described in the previous sections.

Assuming a priori measurement variances (which should also be known quite well after frequent visits to a station), the ratios $\Delta m_i/\sigma_i$ are squared and summed. This test is much more sensitive than the test on the residuals in that it takes into account more than measurement inconsistency; in addition, it tests measurement accuracy.

The test is a χ^2 test as follows:

$$\text{swd} = \sum_{i=1}^{n} \frac{\Delta m_i^2}{\sigma_i^2} = \chi_n^2$$

where swd is pronounced "the sum of the squared, weighted differences."

The positioning evolution is acceptable if the calculated swd does not exceed the standard, S_m, which is found in χ^2 tables at various confidence levels, α .

In the same way s_{Top}^2 is related to s^2, the sum of the squares of the gradient weighted differences, gwd, is related to swd. The formula for gwd is,

$$\text{gwd} = \sum_{i=1}^{n} \frac{G_i^2 \ \Delta m_i^2}{G^2 \ \sigma^2} = \chi_n^2 \qquad \text{where } S_m = \chi_{n,\alpha}^2$$

Figures D-11 and D-12 provide cumulative frequency histograms for Δm and $G\Delta m$ from all REDWOOD fixes studied. The average values are:

SCALE	$G\ \Delta m$	Δm
≤20	13.8 meters	16.3 minutes
>20	20.1 meters	13.9 minutes

D.7.2 Special Case for Difference Between Observed Measurements and Computed Measurements; Example Use of Standards

One of the most frequently employed procedures used by the Coast Guard in positioning aids is called the fixed glass procedure. The fixed glass procedure is performed by setting two sextants at prescribed angles and maneuvering the ship until both measurements agree with the prescribed angles. Presently, this procedure often includes only two measurements and should be extended to satisfy the requirement for at least three measurements. To do this, the positioning team continually makes a third measurement, thus allowing a continuous check on the consistency of the measurement set. The problem is to define the limits within which the third measurement must lie so that the measurement set, as a whole, indicates an adequate positioning effort. The derivation of the relationship that exists between the third measurement difference, Δm_3, and resulting position errors is as follows.

The fixed glass method required that two observed measurements agree with the corresponding computed measurements. Let the third observed measurement differ from the corresponding computed measurement by some value Δm_3 and still indicate an adequate positioning effort. That is, some measure of success S is a function of Δm_3 and the standard S_m, corresponds to a limit on Δm_3. As an example, the major semi-axis, A_s, is found as a function of Δm_3, start with equations (C-18) and (C-13) with (n=3).

$$A_s = M_s A_1 s$$

$$s^2 = \frac{\underline{L}^T \underline{\underline{W}}\underline{L} - \underline{L}^T \underline{\underline{W}}\underline{\underline{A}}\underline{X}}{n-2} = \underline{L}^T \underline{\underline{W}}\underline{L} - \underline{L}^T \underline{\underline{W}}\underline{\underline{A}}\underline{X}$$

Because two measurements are on the mark, only one Δm_i remains. Which one depends on which of the three is the third measurement. s_i^2 depends on Δm_i as follows:

$$s_i^2 = \Delta m_i w_{ii} \Delta m_i - \Delta m_i w_{ii} \left[a_{i1} \quad \Delta x + a_{i2} \quad \Delta y\right]$$

Using the summation notation of equation (B-3), Δx and Δy are:

$$\Delta x = \frac{BE-CD}{AE-C^2}$$

$$\Delta y = \frac{AD-BC}{AE-C^2}$$

s_i^2 is

$$s_i^2 = \Delta m_i w_{ii} \quad \Delta m_i - \Delta m_i w_{ii} \left[\frac{a_{i1}(BE-CD)}{AE-C^2} + \frac{a_{i2}(AD-BC)}{AE-C^2}\right]$$

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For this case, B and D can be reduced to

$$B = \Delta m_i a_{i1} \qquad D = \Delta m_i a_{i2}$$

and,

$$s_i^2 = \Delta m_i^2 w_{ii} - \frac{\Delta m_i w_{ii}}{AE-C^2}\left[a_{i1}\,(a_{i1}\,\Delta m_i E - C a_{i2}\,\Delta m_i) + a_{i2}(A\,\Delta m_i a_{i2} - C a_{i1}\,\Delta m_i)\right]$$

But,

$$a_{i1} = \frac{\sin\ \gamma_i}{G_i} \qquad a_{i2} = \frac{\cos\ \gamma_i}{G_i}$$

and finally,

$$s_i^2 = \Delta m_i^2 w_{ii}\left[1 - \left(\frac{A\cos^2\gamma_i + E\sin^2\gamma_i - 2C\sin\gamma_i\cos\gamma_i}{G_i^2\ (AE-C^2)}\right)\right]$$

The major semi-axis is

$$A_s = M_s A_1\,\Delta m_i (w_{ii})^{1/2}(1-k_i)^{1/2}$$

where

$$k_i = \frac{A\cos^2\gamma_i + E\sin^2\gamma_i - 2C\sin\gamma_i\cos\gamma_i}{G_i^2\ (AE-C^2)}$$

The minor semi-axis is

$$B_s = M_s B_1\,\Delta m_i (w_{ii})^{1/2}(1-k_i)^{1/2}$$

and the confidence ellipse area is

$$Area_s = M_s^2 A_1 B_1\,\Delta m_i^2 w_{ii}(1-k_i) = \frac{M_s^2\ \Delta m_i^2 w_{ii}(1-k_i)}{(AE-C^2)^{1/2}}$$

The approximating circle of confidence, COC, is

$$COC = M_S \sqrt{A_1^2 + B_1^2} \quad \Delta m_i (w_{ii})^{1/2} \ (1-k_i)^{1/2}$$

and finally the semi-diameter of the confidence ellipse in some direction δ, clockwise from geodetic north is given by

$$A_S = \frac{M_S \ \Delta m_i (w_{ii})^{1/2} \ (1-k_i)^{1/2}}{\left(\frac{\sin^2(\delta - \beta_{CE})}{A_1^2} + \frac{\cos^2(\delta - \beta_{CE})}{B_1^2} \right)^{1/2}}$$

where β_{CE} is the orientation of the confidence ellipse clockwise from geodetic north.

The results of this derivation shows the linear relationship that exists between the measurement difference, Δm_3, and the major semi-axis. The slope of the linear function is dependent on the confidence level desired, the weight of the third measurement relative to the two other measurements and the geometry of the fix.

In the following example, the major semi-axis is calculated as a function of the third measurement difference for a geometry which consists of three sextant measurements of equal weight and which determine LOPs with γ_i=(0,60,120) G_i=1.0. The 90% confidence level ellipse major semi-axes is calculated producing the following results:

$$A_S = M_S A_1 \Delta m_i (w_{ii})^{1/2} (1-k_i)^{1/2}$$

$$w_{ii} = w_{22} = w_{33} = 1.0$$

$$A = E = 1.5 \qquad k_1 = k_2 = k_3 = 0.67$$

$$C = 0 \qquad A_1 = 0.67$$

$$M_S = \sqrt{2F_{2,1,0.10}} = 9.94$$

Using M_S in the A_S expression, we have

$$A_S = (9.94)(0.67)(1-0.67)^{1/2} \Delta m_i = (3.83 \ \frac{\text{met}}{\text{min}}) \ \Delta m_i$$

The geometry is symmetric about the designated crossing point which means the major semi-axis is not dependent on which of the three measurements is chosen to be the check angle. The maximum acceptable Δm_i

which still indicates an adequate fix is determined by the chosen semi-major axis standard S_m and the proceding equation,

$$\Delta m_i = \frac{S_m}{(3.83 \frac{met}{min})}$$

For example, if a major semi-axis standard of twenty meters is established for the example geometry studied, the first two measurements are caused to coincide by maneuvering the ship and the consistency is adequate if,

$$\Delta m_3 \leq \frac{20 \text{ meters}}{3.84 \frac{met}{min}} = 5.2 \text{ minutes}$$

In other words, the angular measurement must be within 5.2 minutes of the computed measurement. Similar calculations can be performed using other standards; such as, confidence ellipse area, directional semi-diameters.

NOTE: Once the third measurement differences have been calculated using one of the above equations, they can be used anywhere on the grid diagram to check measurement consistency. This procedure is as follows:

> *On the grid diagram*, use dividers to find the distance in minutes from the point where two lines of position intersect to the third line of position (along a perpendicular line). The result must be less than the standard set for the third measurement difference.

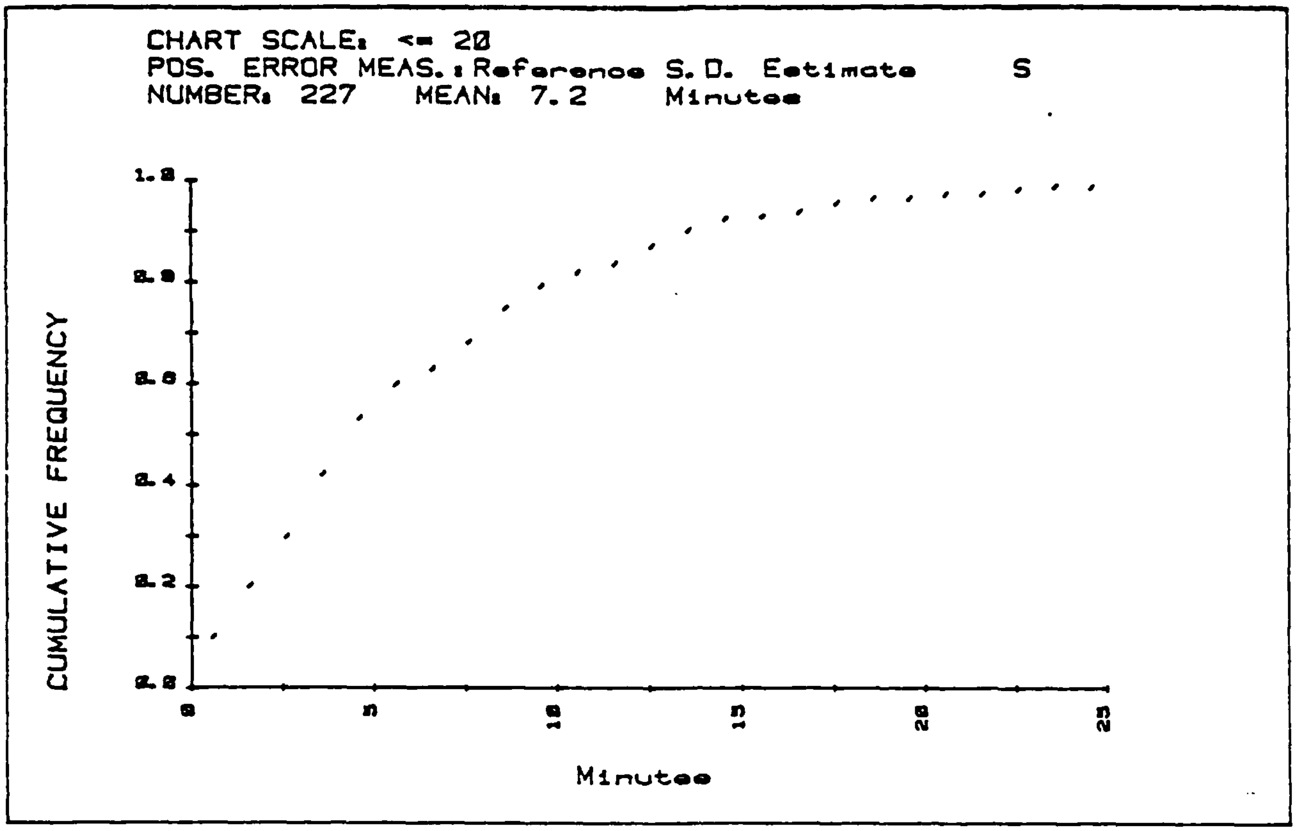


FIGURE D-1(a)

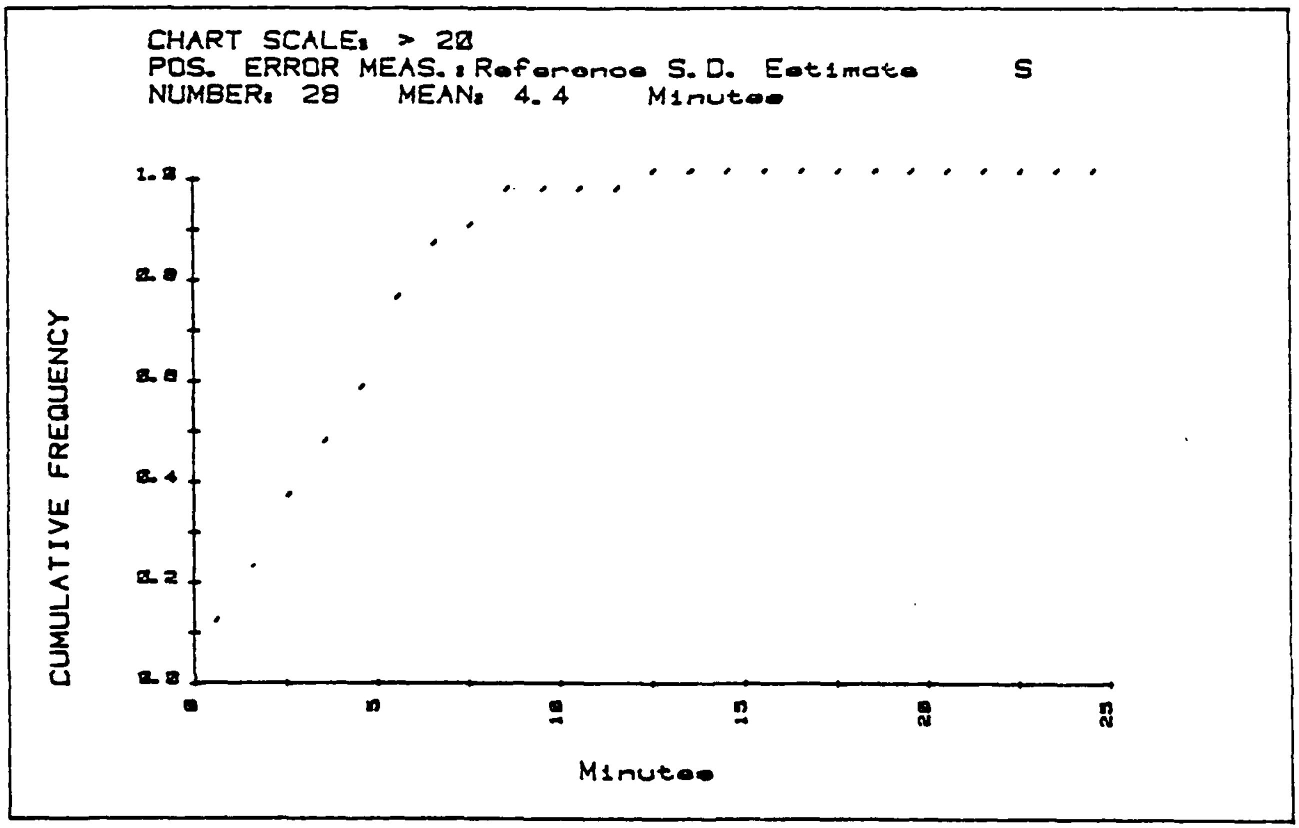


FIGURE D-1(b)

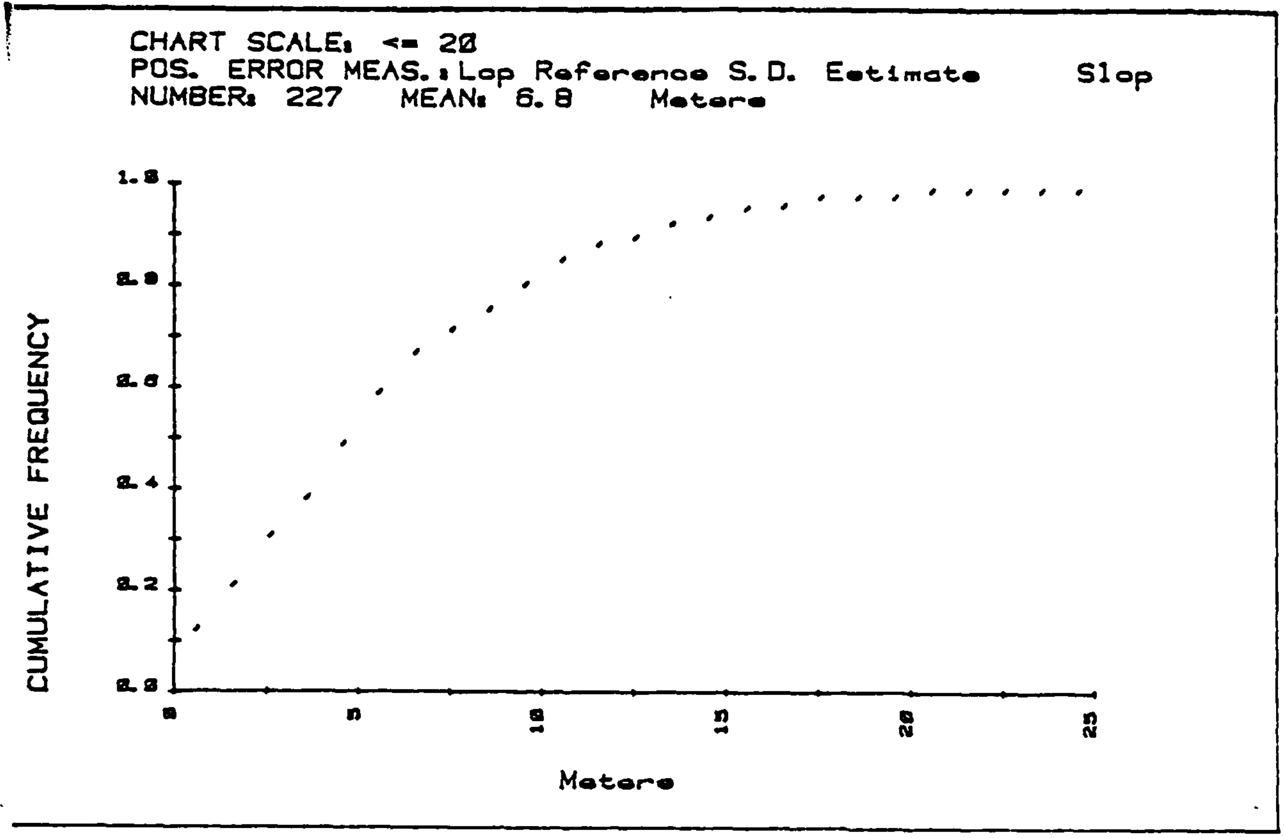


FIGURE D-2(a)

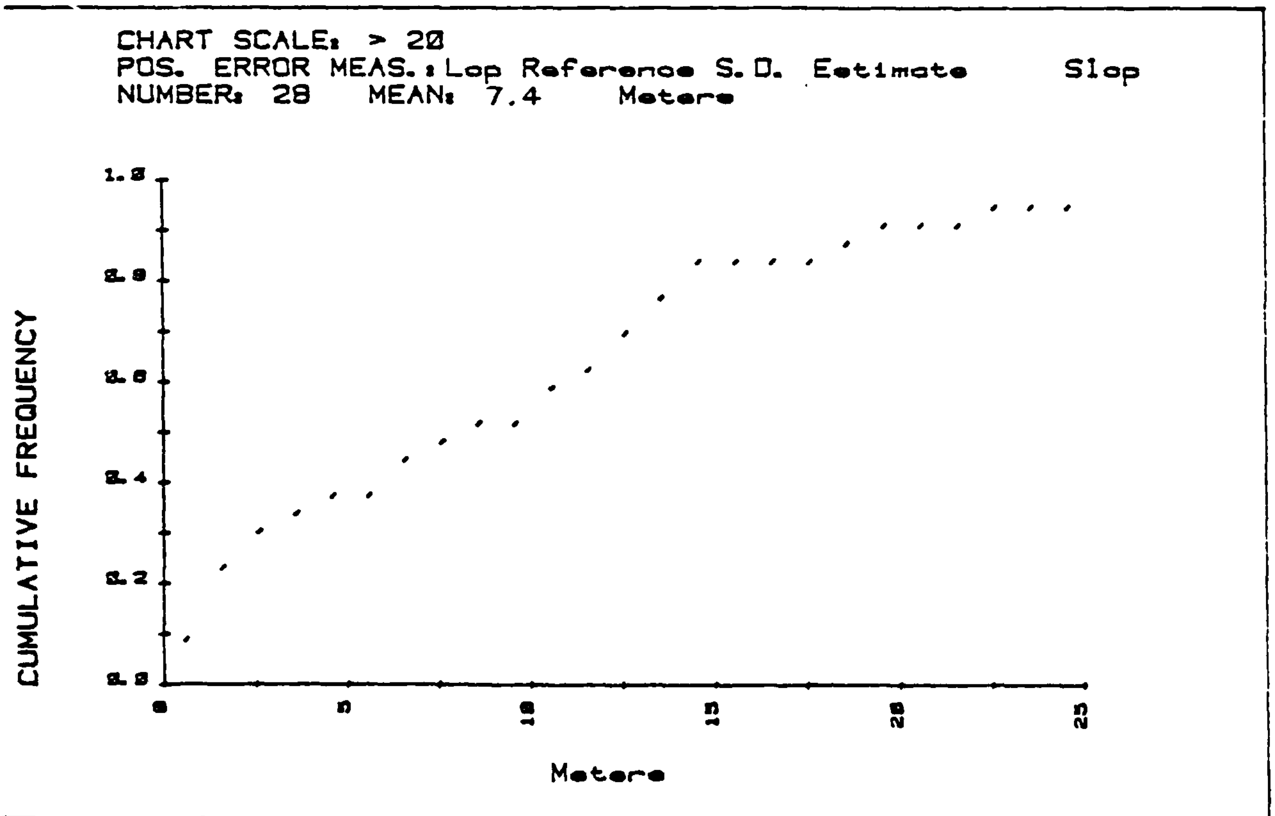


FIGURE D-2(b)

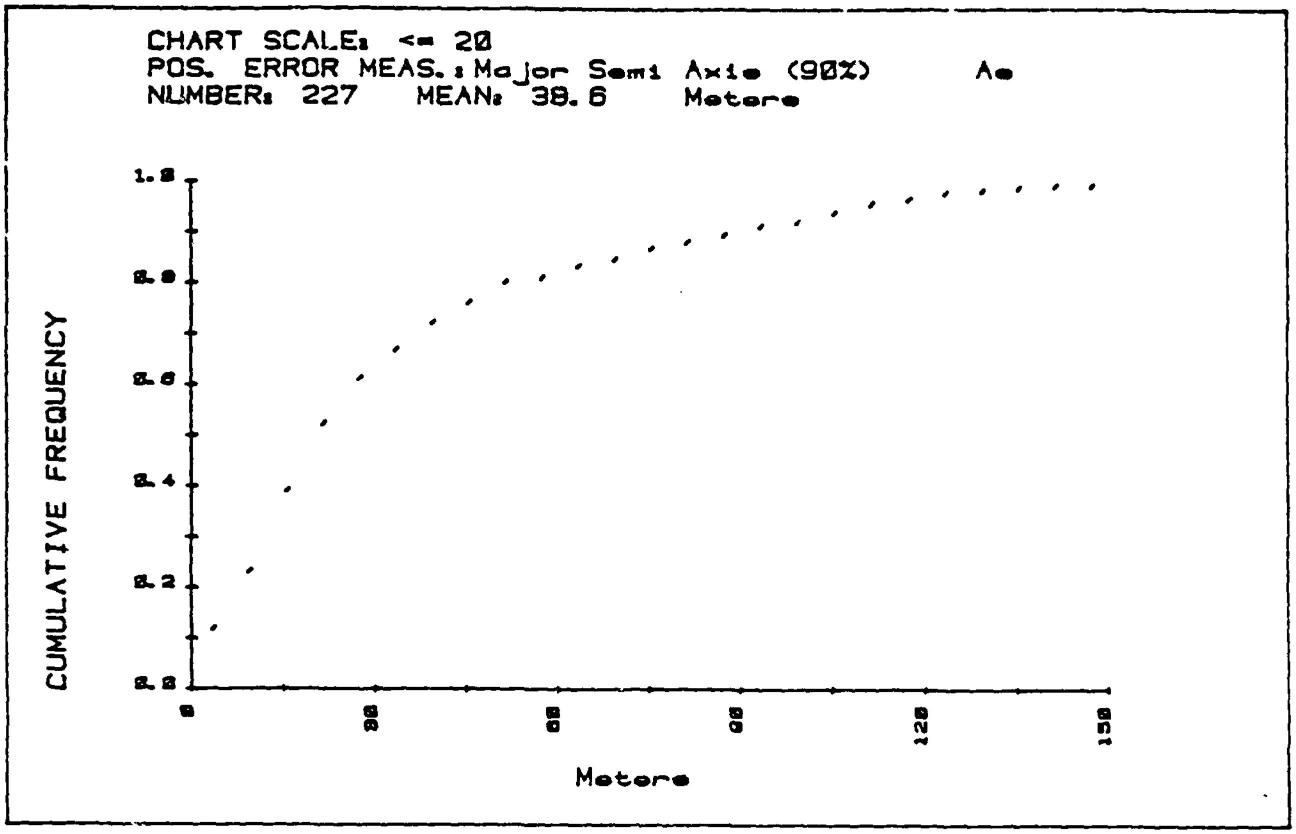


FIGURE D-3(a)

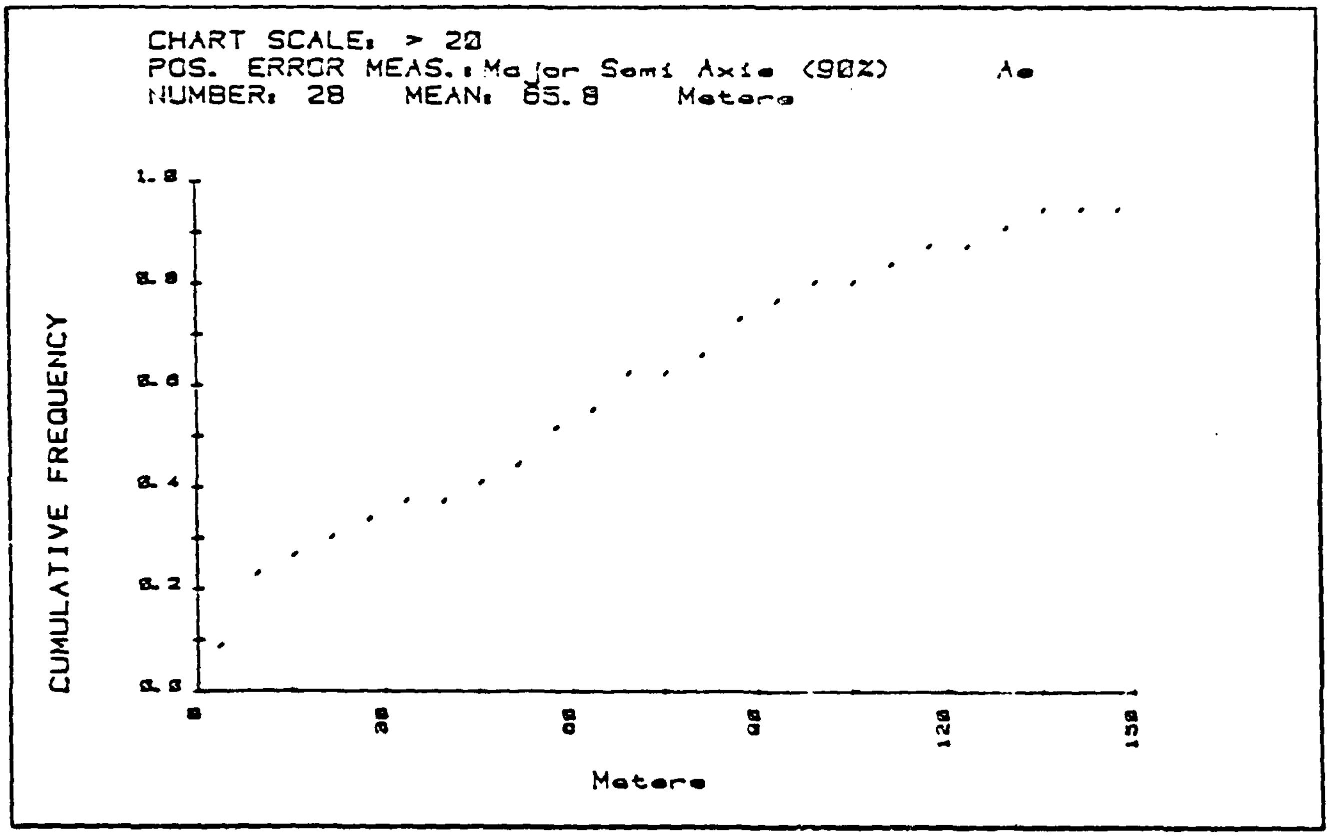


FIGURE D-3(b)

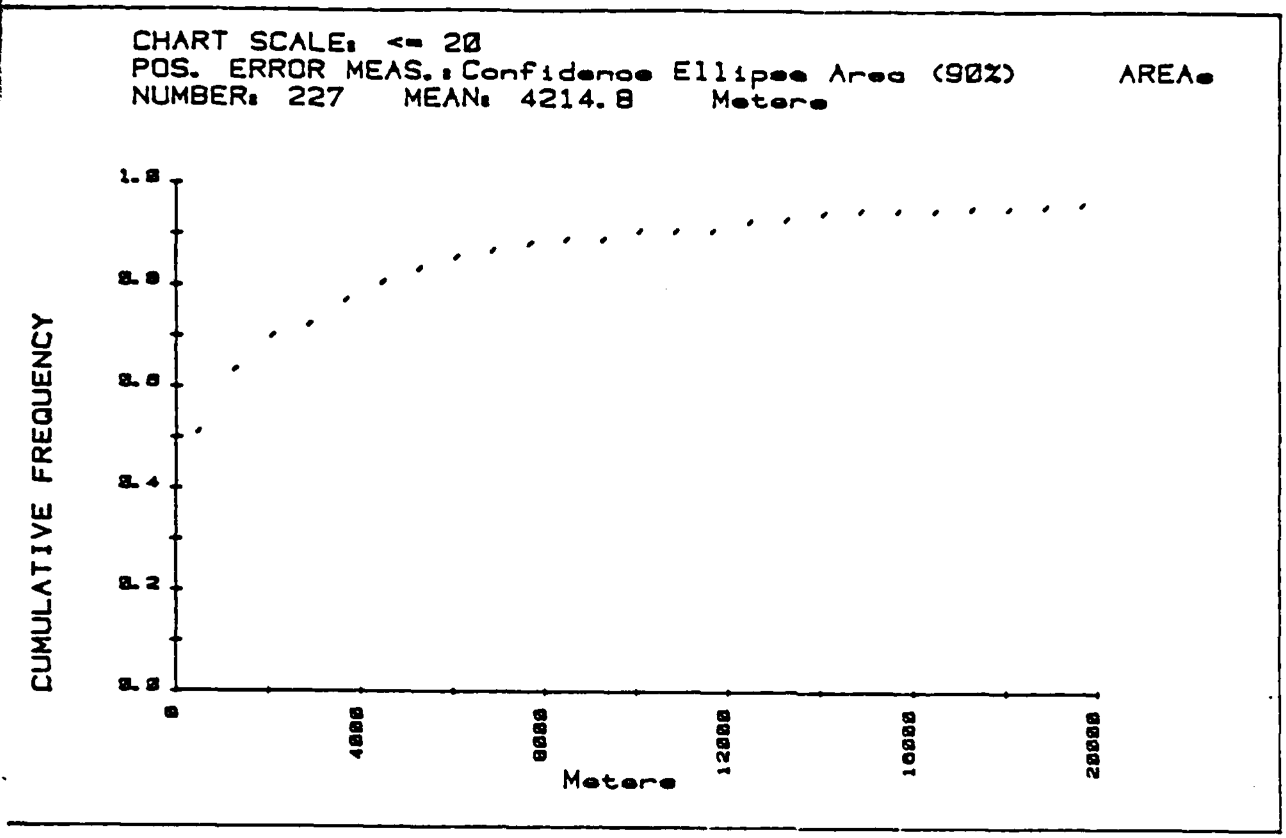


FIGURE D-4(a)

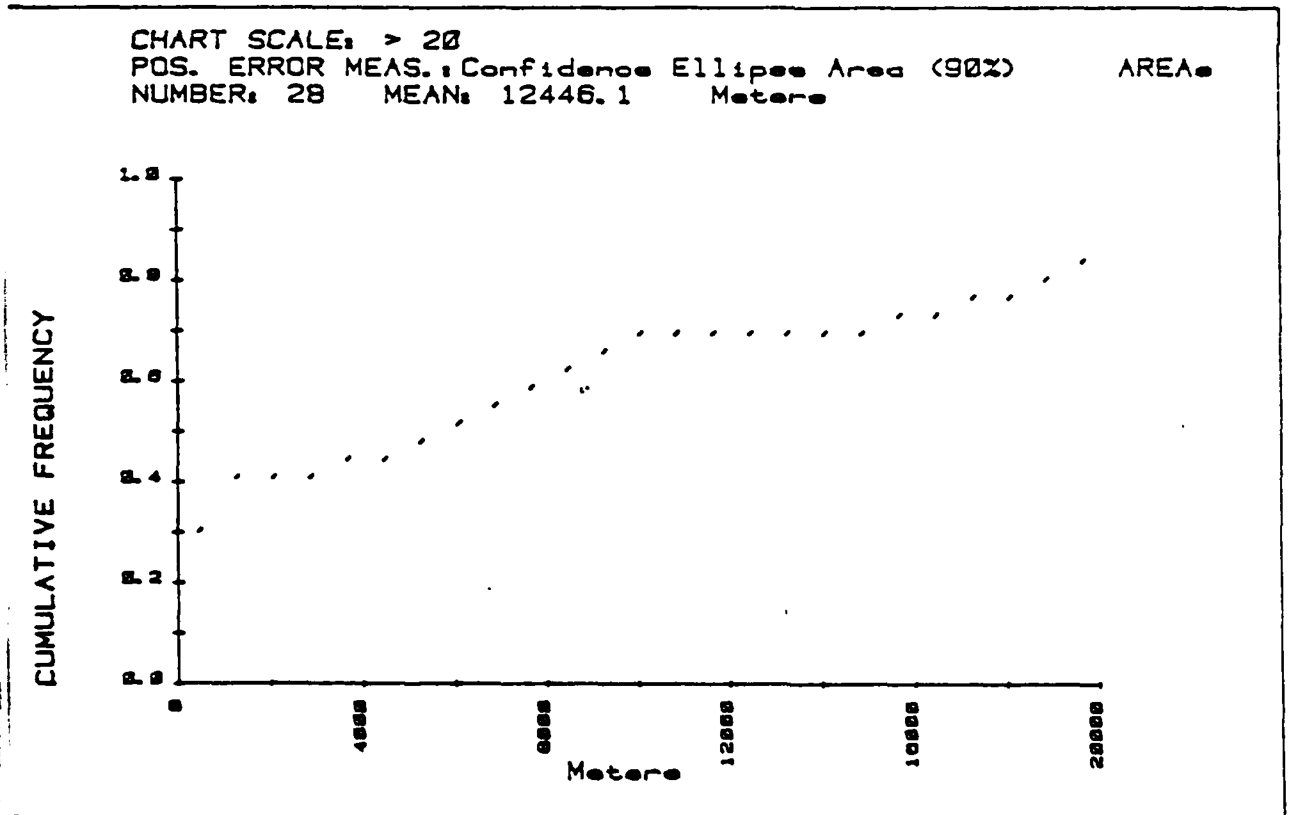


FIGURE D-4(b)

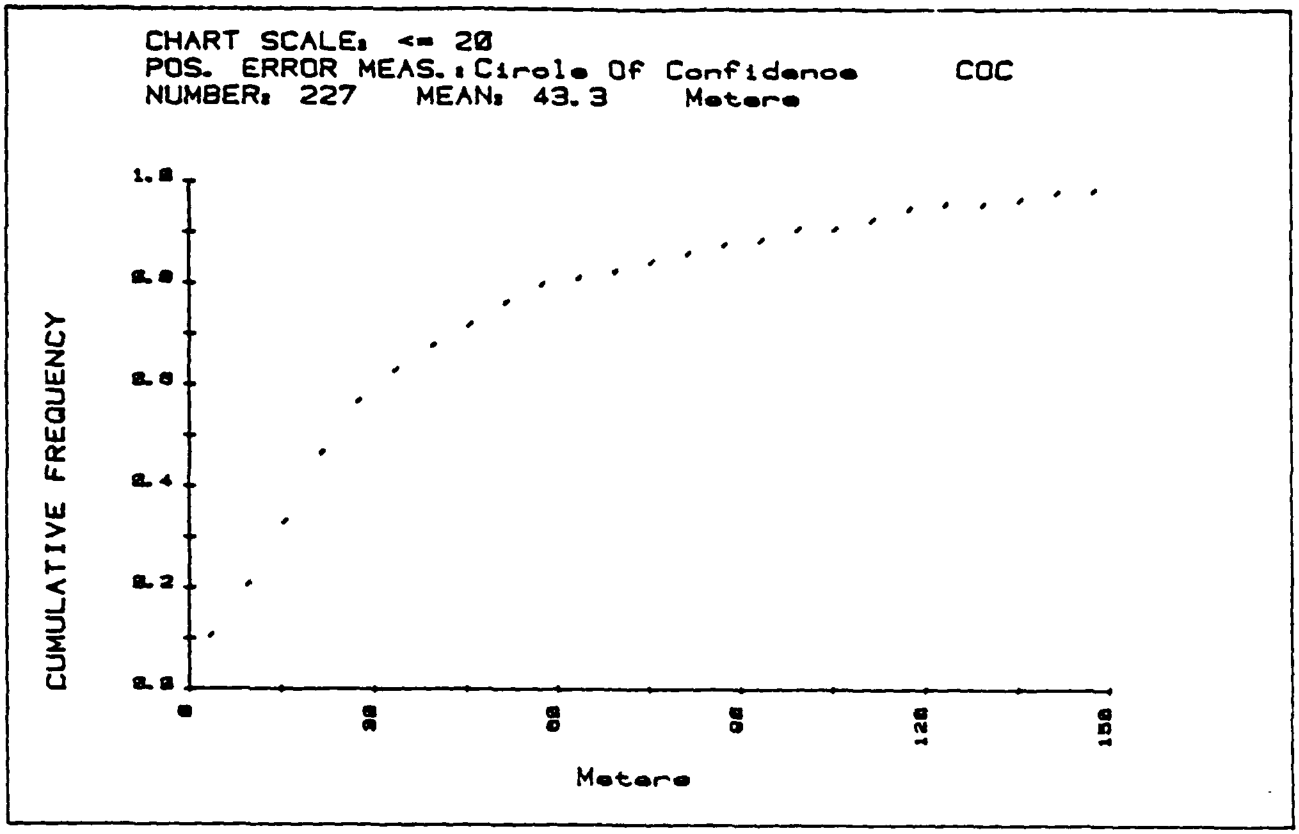


FIGURE D-5(a)

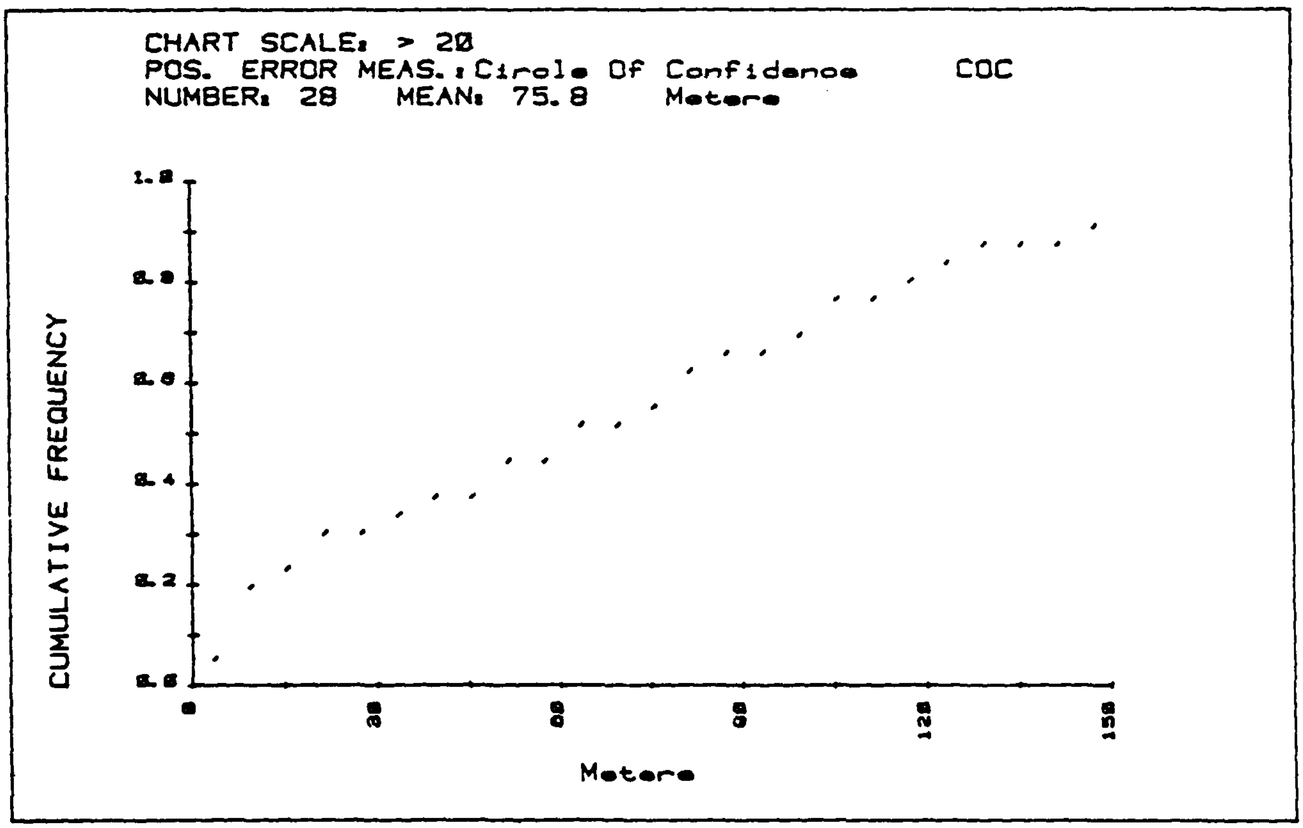


FIGURE D-5(b)

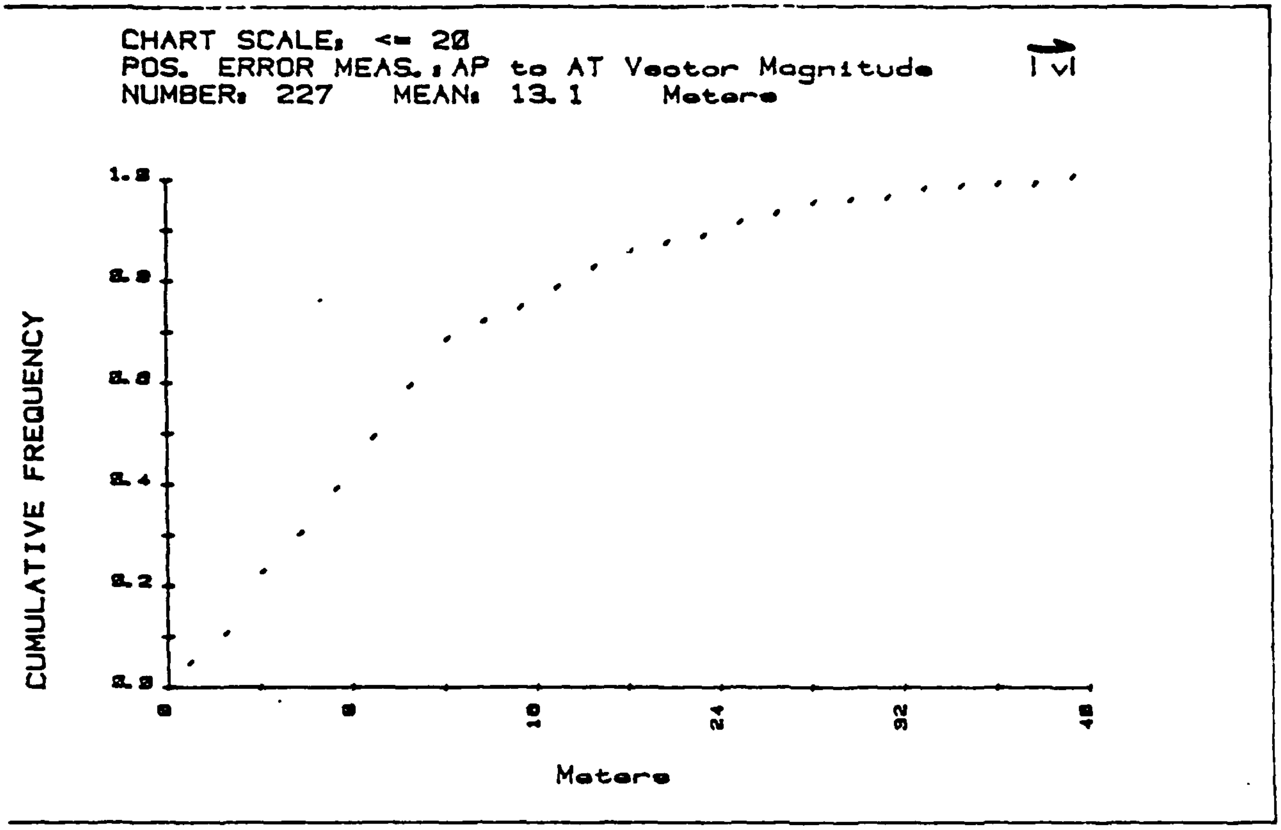


FIGURE D-6(a)

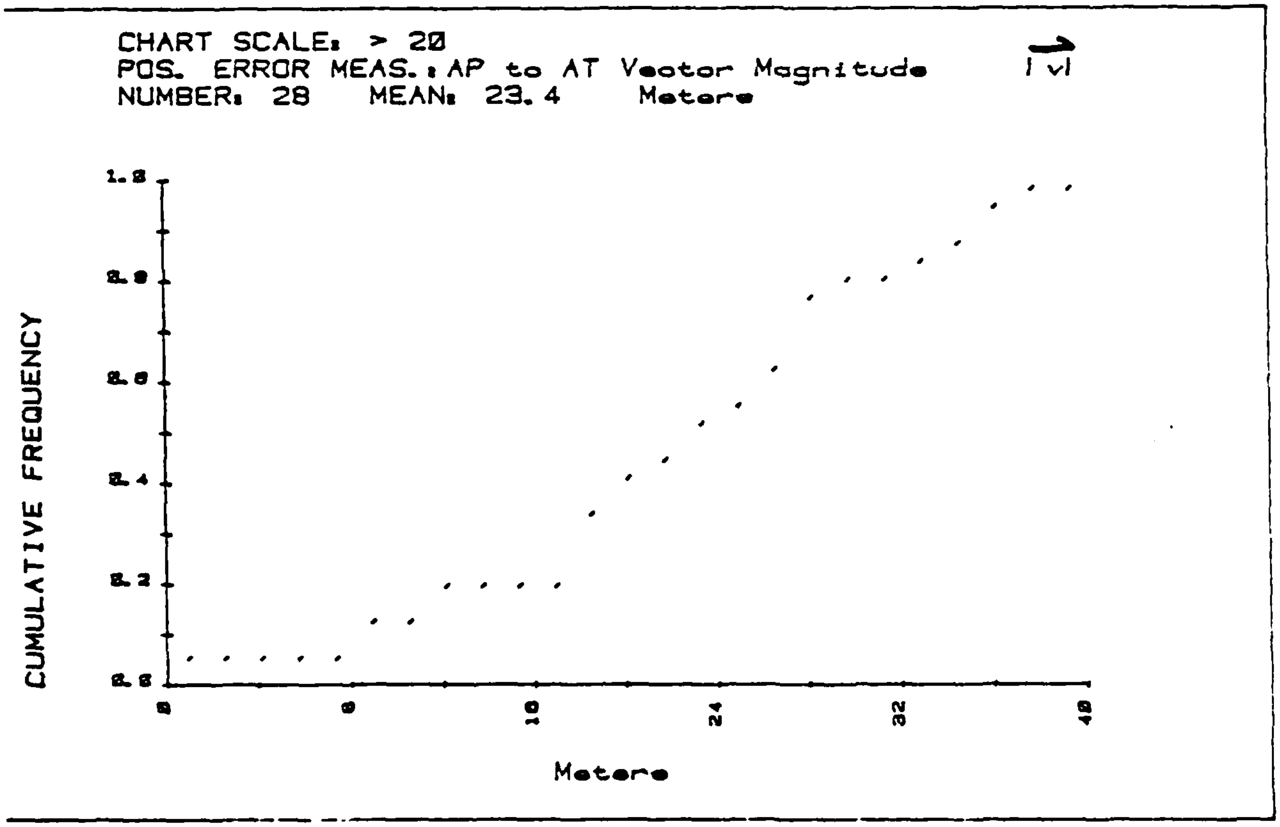


FIGURE D-6(b)

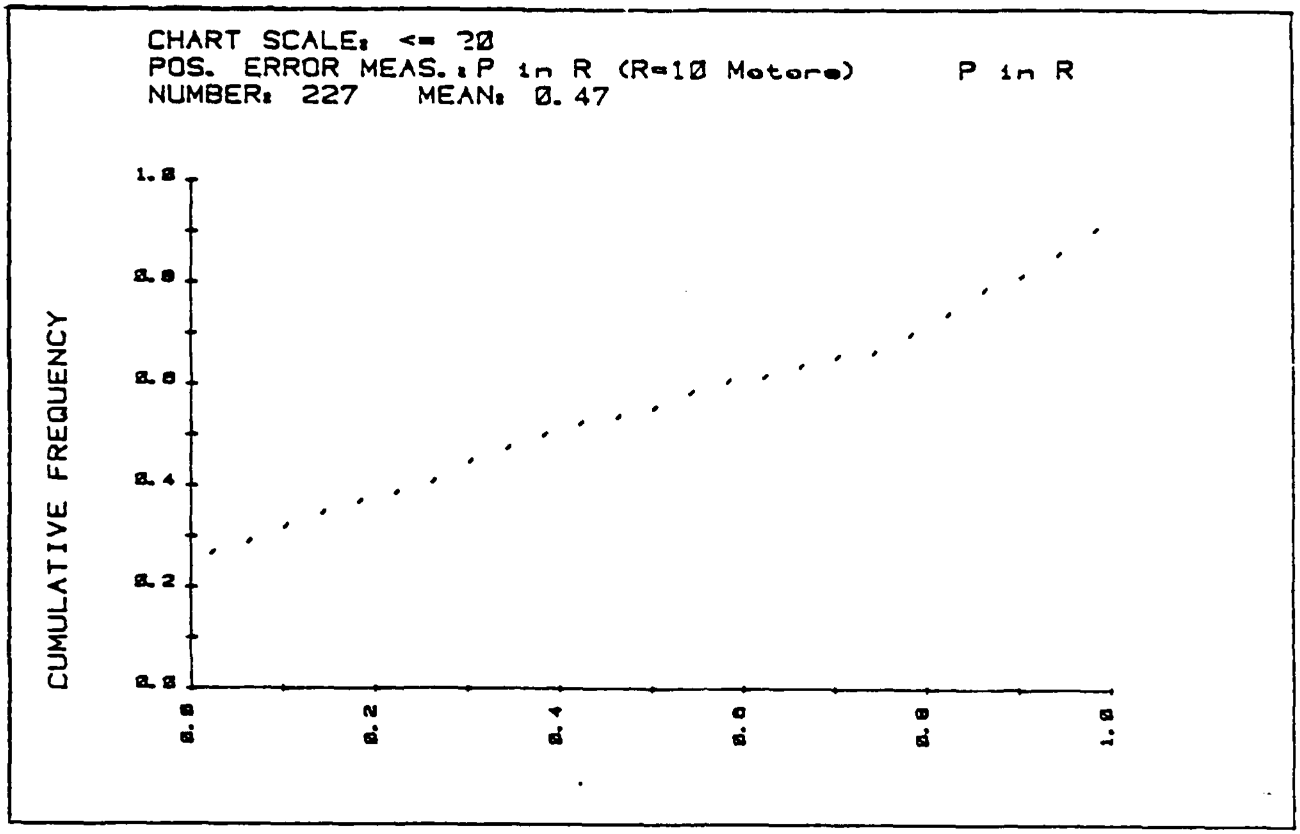


FIGURE D-7(a)

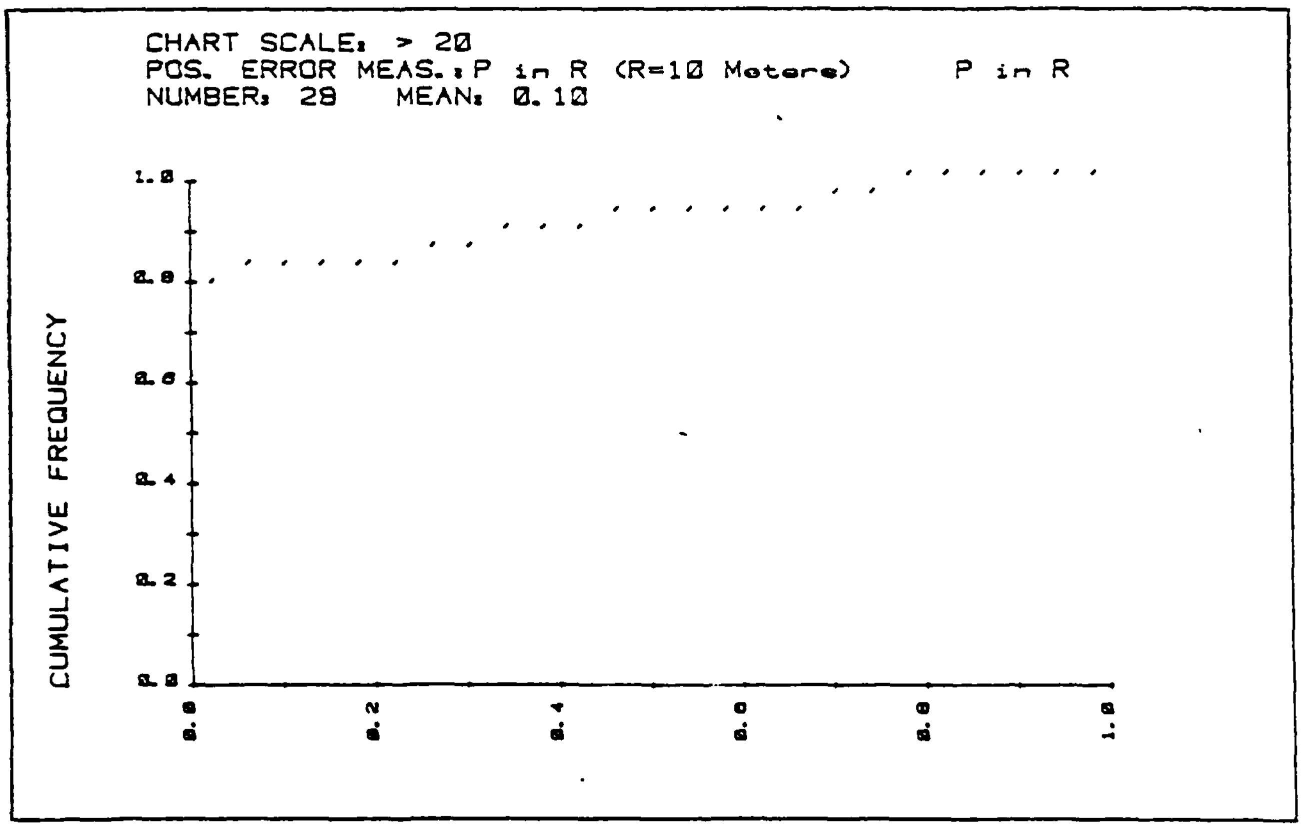


FIGURE D-7(b)

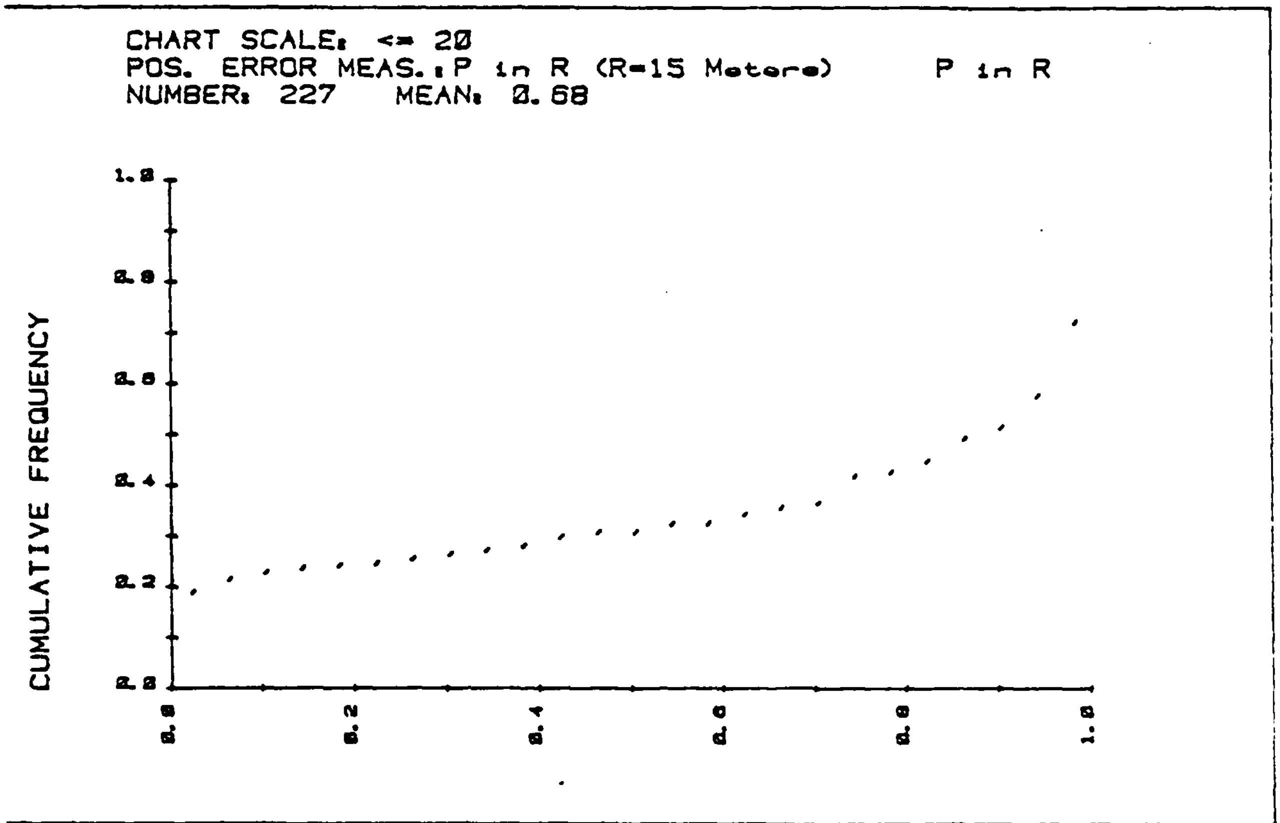


FIGURE D-8(a)

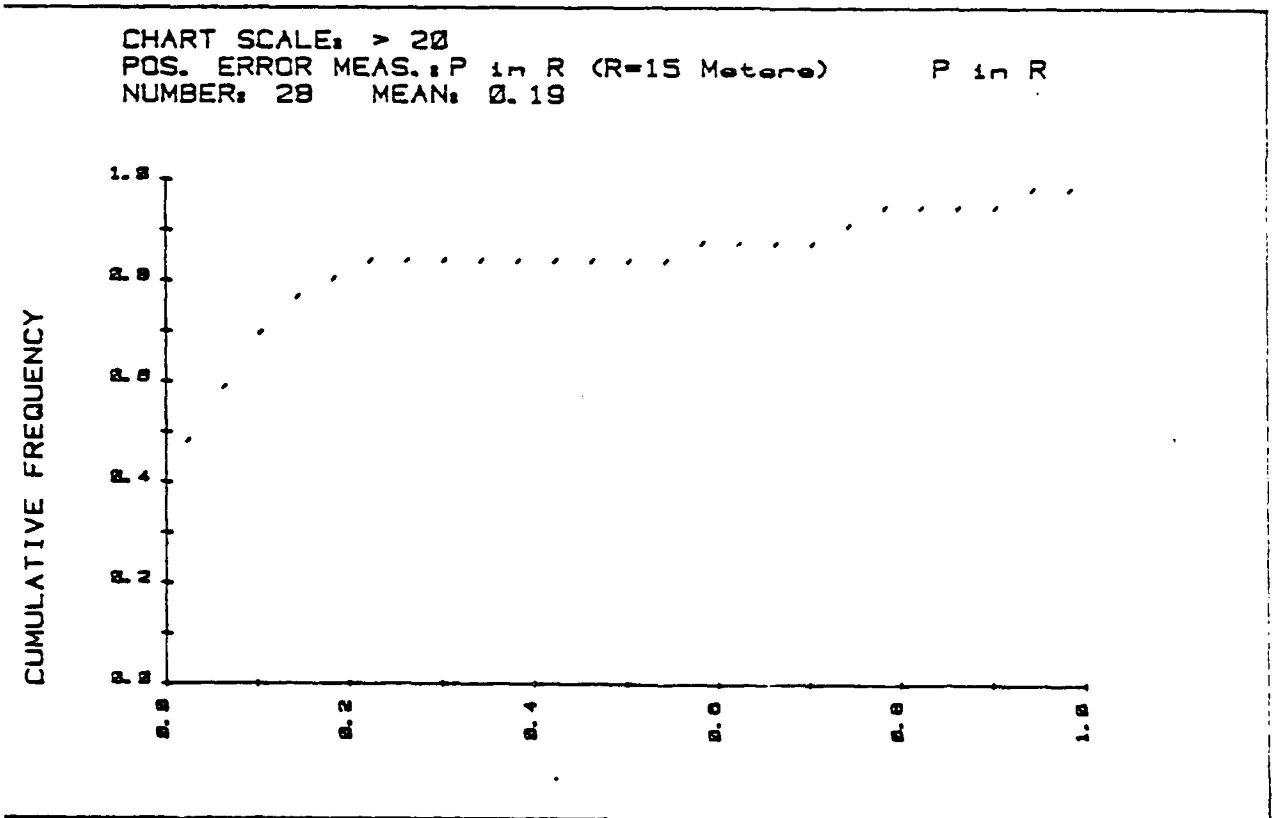


FIGURE D-8(b)

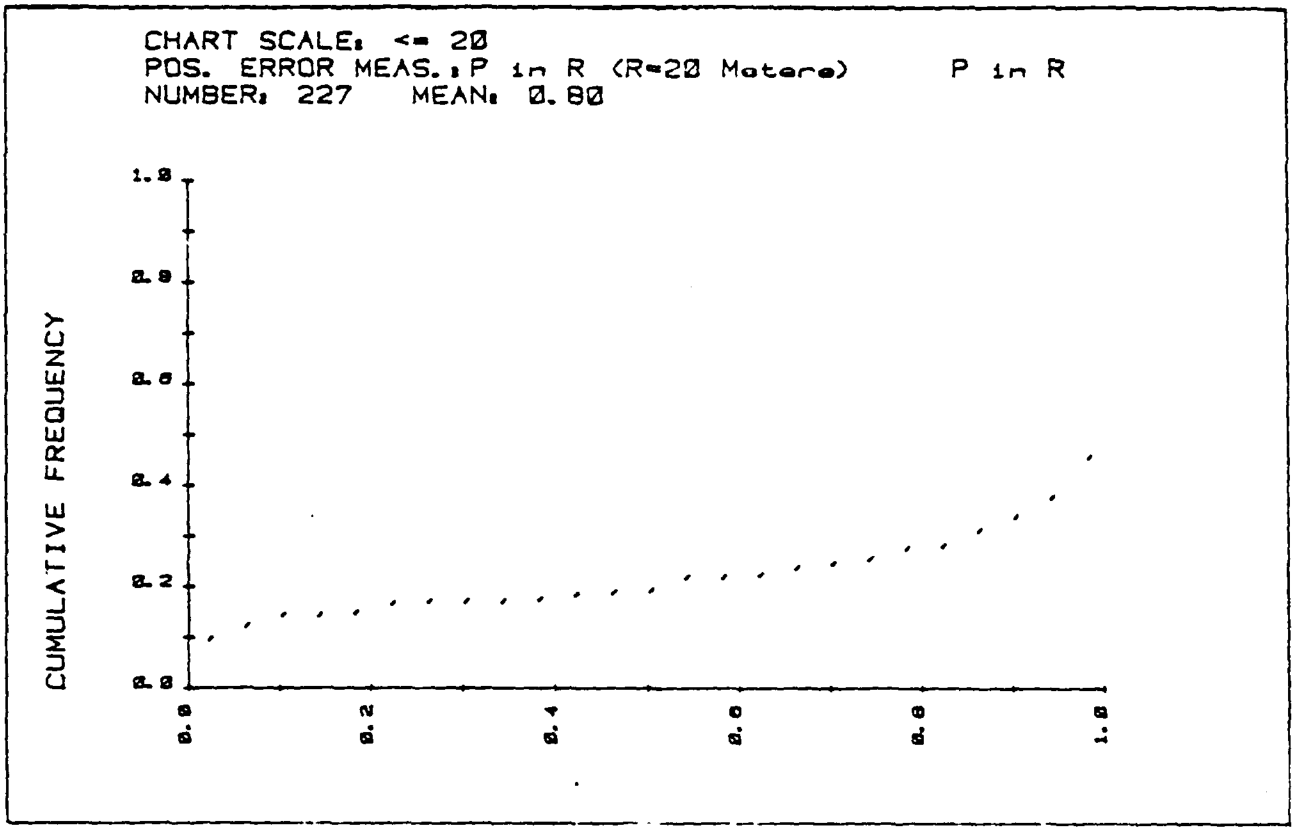


FIGURE D-9(a)

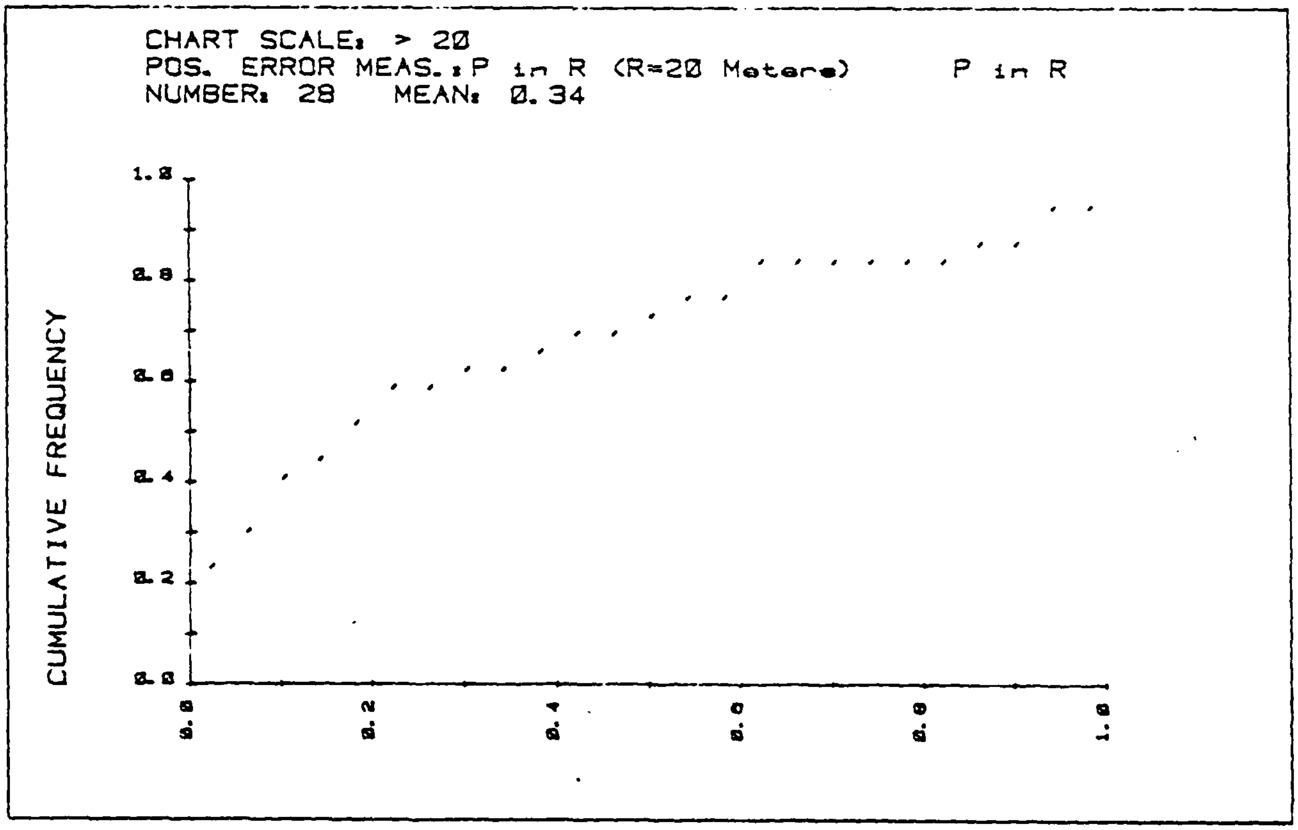


FIGURE D-9(b)

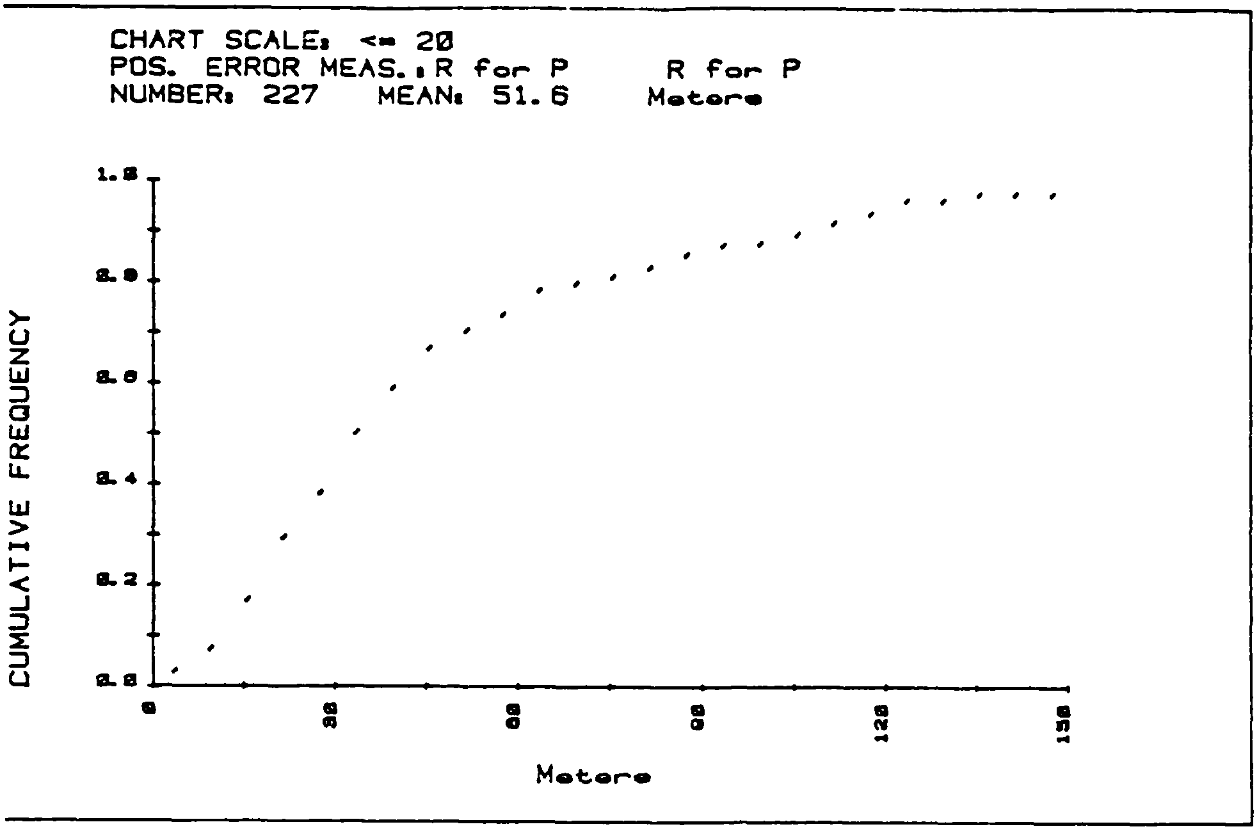


FIGURE D-10(a)

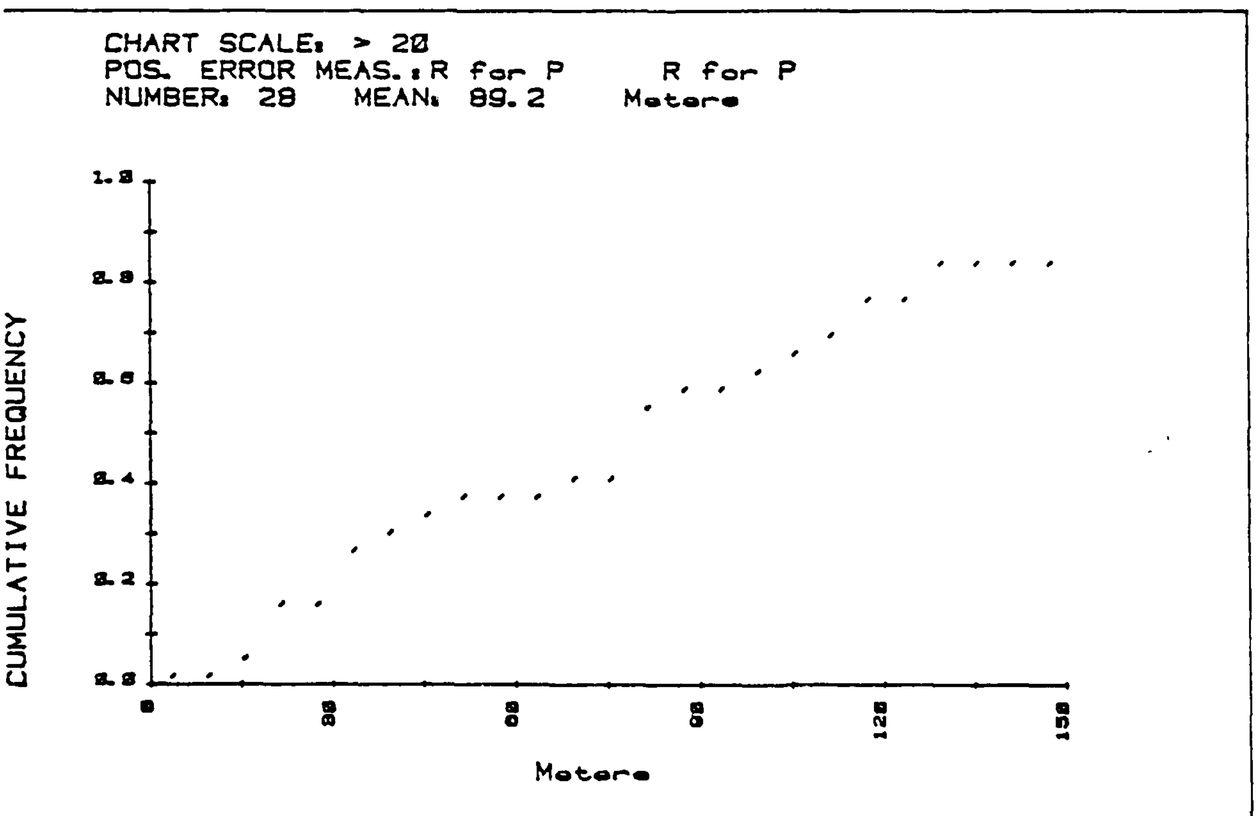


FIGURE D-10(b)

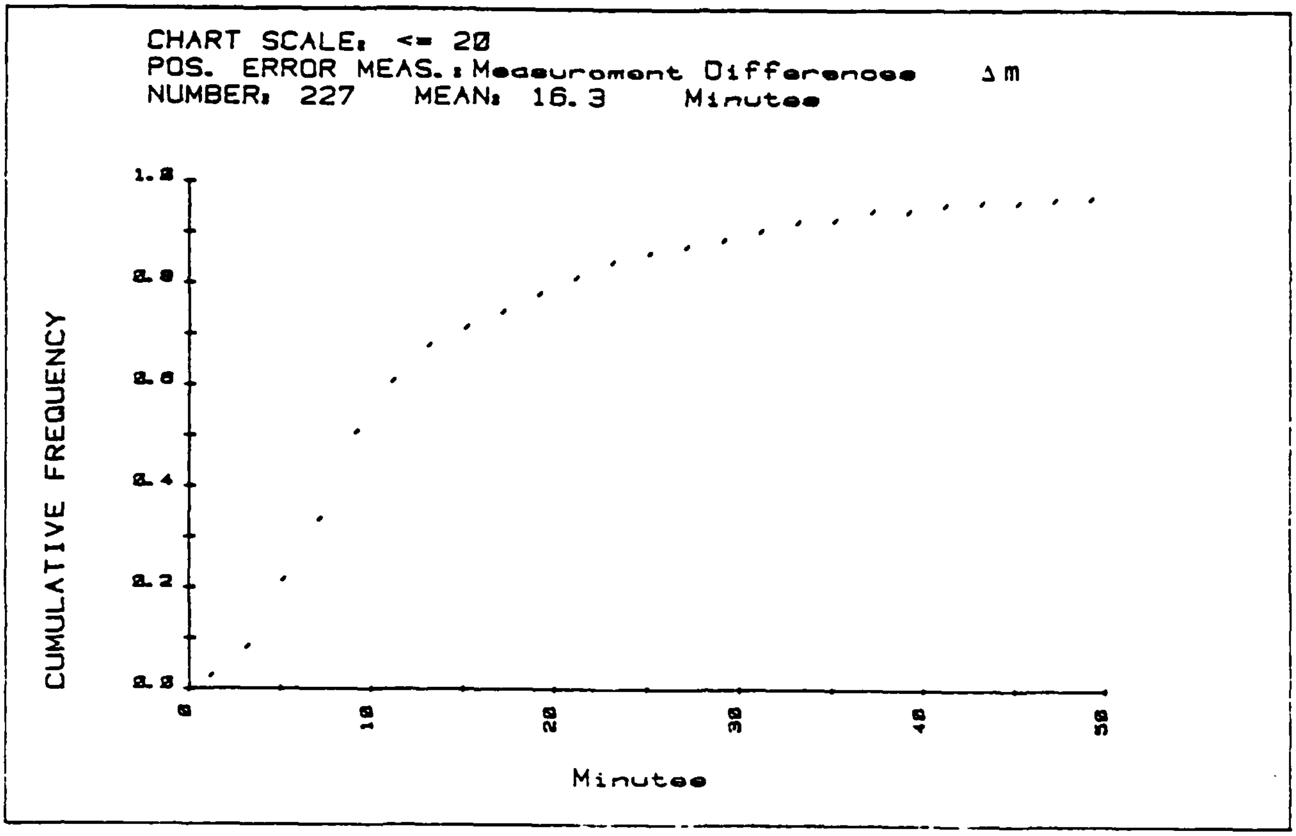


FIGURE D-11(a)

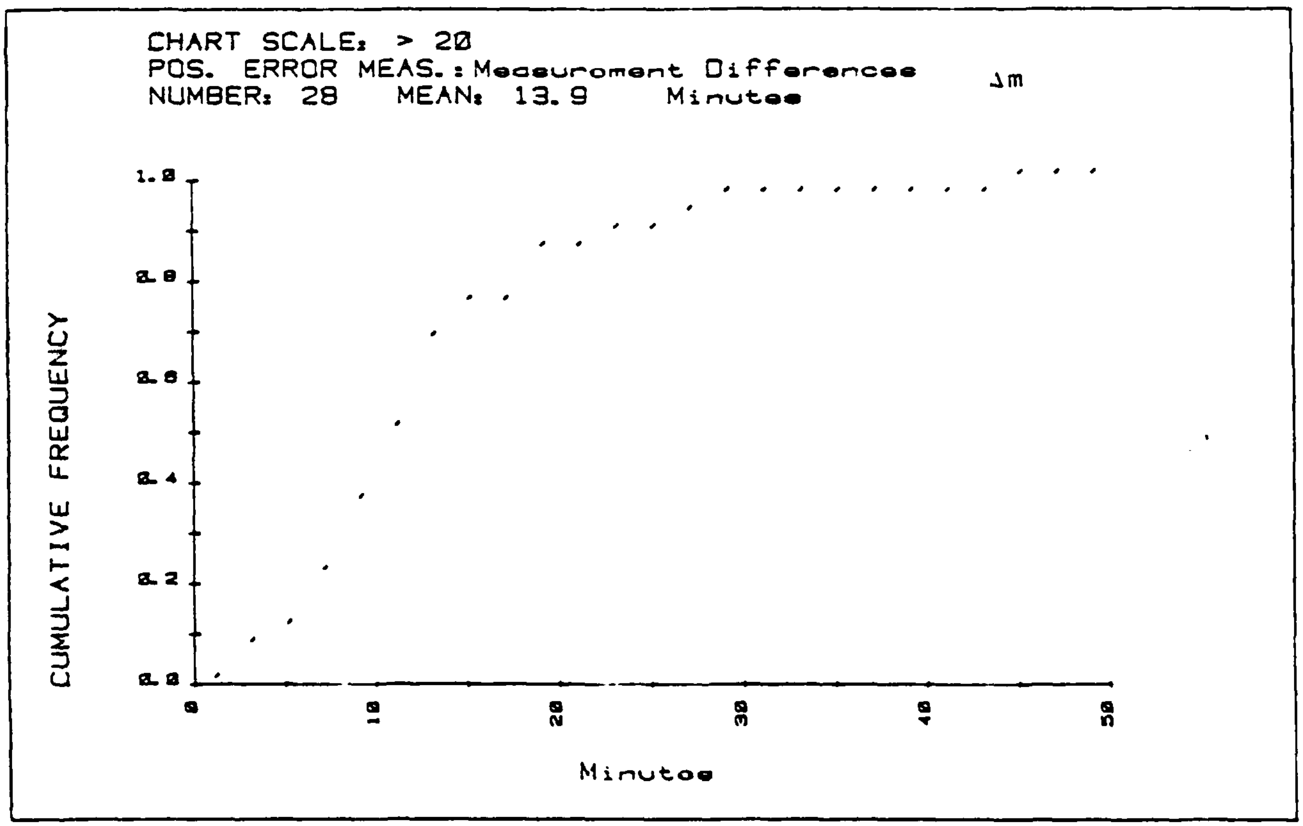


FIGURE D-11(b)

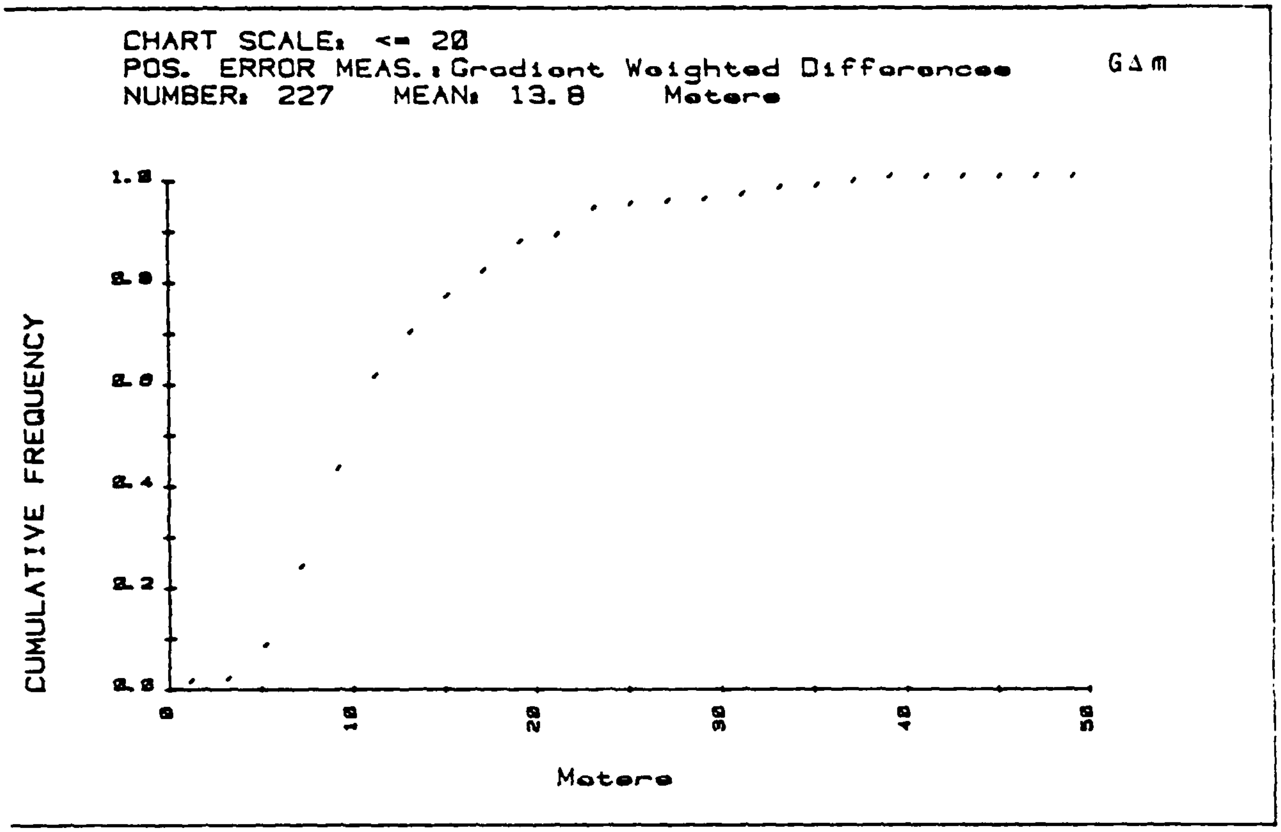


FIGURE D-12(a)

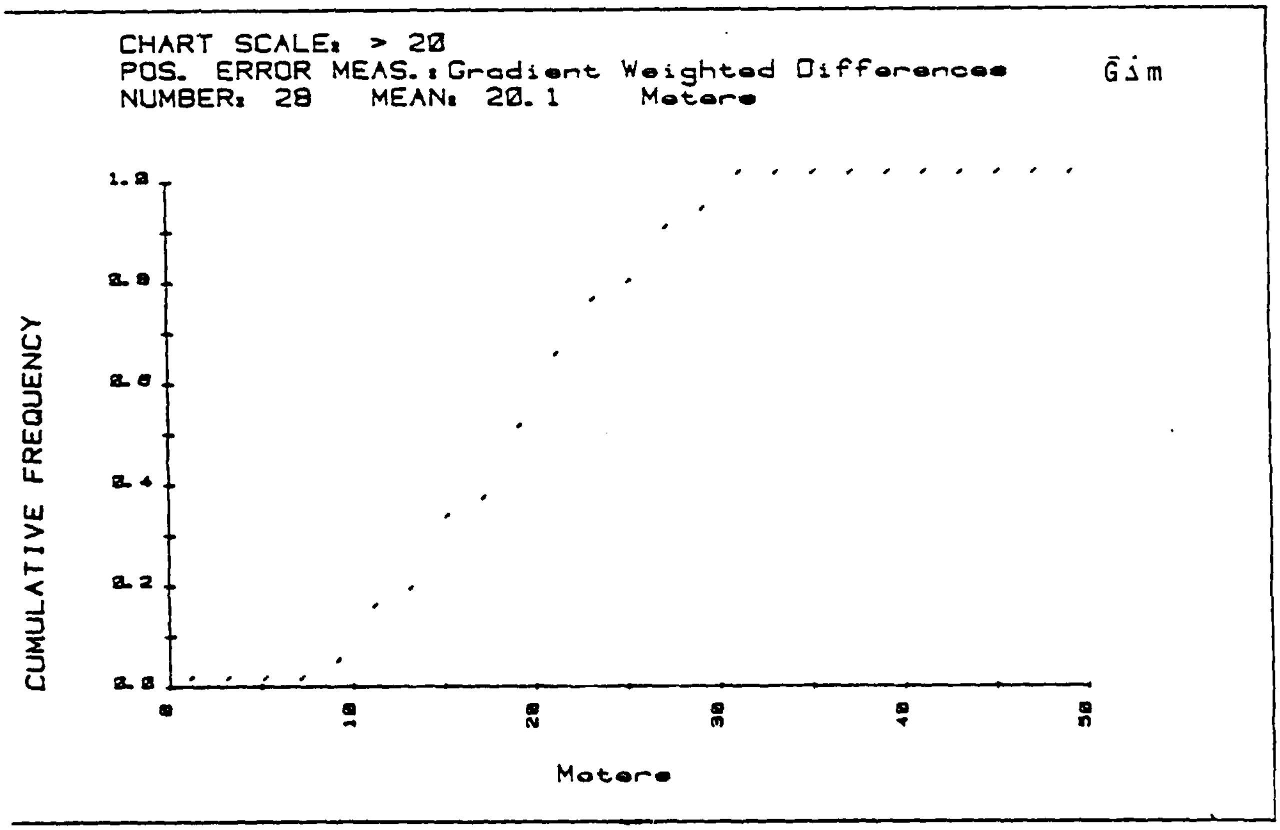


FIGURE D-12(b)

APPENDIX E

DETECTION OF MEASUREMENT OUTLIERS

E.1 A POSTERIORI REFERENCE VARIANCE ESTIMATE

The use of s^2 in detecting outliers is similar to its use as a position error measure. In appendix D.1, the statistical tests are outlined for comparing s^2 to expectations. Rejection of the hypothesis that $\sigma^2 = \sigma_0^2$ should lead to serious review of the data for blunders.

E.2 MEASUREMENT COMBINATIONS

In any case where more than three measurements are made in positioning, a statistical analysis of various combinations of the measurements can be performed with the goal of detecting a measurement which is an outlier at some level of confidence. This method was alluded to by Rosenblatt (reference 12) and outlined in reference 2, appendix F.

The analysis begins with a set of n inconsistent measurements at a predesignated confidence level α. This is indicated by the test

$$\frac{(n-2)\, s^2}{\sigma_0^2} > \chi^2_{n-2}, \tag{E-1}$$

The problem is to identify which of the n measurements are causing the inconsistency. The inconsistency may not be due to only one poor measurement; however, it is logical to test for this possibility first. The approach is to temporarily remove one measurement from the set of n measurements $\binom{n}{1}$ times thereby creating n different reduced fixes. The reduced a posteriori estimate of the reference variance from each of the $\binom{n}{1}$ reduced fixes has n-3 degrees of freedom. One degree of freedom lies with the measurement residual of the removed measurement. This residual is calculated by comparing the actual measurement with that measurement which would have been made if the observer were at the reduced MPP. There are now $\binom{n}{1}$ reduced estimates of the reference variance and an equal number of removed residuals. The removed residual is weighted by the corresponding element of the weighting matrix, $\underline{W}$, which allows the following ratio to be calculated $\binom{n}{1}$ times,

$$\frac{\left(\dfrac{r_i^2}{\sigma_0^2}\right)}{\dfrac{(n-3)\, s_R^2}{\sigma_0^2}} = \frac{\chi_1^2}{\chi_{n-3}^2} \quad \text{for } i = 1 \text{ to } n \tag{E-2}$$

The numerator has one degree of freedom, the denominator has n-3 degrees of freedom. The largest of the $\binom{n}{1}$ ratios is tested against the F statistic at some level α. Thus

$$\left[\frac{r_i^2 \, w_{ii}}{s_R^2}\right]_{max} > F_{1,n-3,\alpha} \qquad \text{(E-3)}$$

The hypothesis (that the maximum ratio is not excessive) is rejected if the calculated ratio exceeds the critical $F_{1,n-3,\alpha}$ value. In any case where the hypothesis is rejected, the corresponding measurement (that was temporarily removed for testing) can be rejected. The remaining n-1 measurements can be further tested for compliance with any applicable standards, such as against the new χ^2_{n-2} value (n) now being reduced by one from that in equation (E-1).

As mentioned earlier, there is no guarantee that only one measurement of the n measurements is unacceptable. In fact all measurements can cumulatively contribute to an unacceptable inconsistency in the measurements. The probability that all measurements are in large error is small; but one can easily envision scenarios where two measurements contribute to the inconsistency. For example, suppose that a rebuilt but not resurveyed landmark is sighted in two separate sextant angular measurements. Though the measurements are statistically independent, there is a high probability that they are mutually inaccurate. The problem is to statistically identify two inaccurate measurements and (assuming they cannot be corrected) identify them from the measurement set. It is obvious at this point that the number of measurements required to perform statistical outlier detection is quite large. If one were to attempt to identify two statistical outliers from a set of only four measurements, only two-measurement subsets remain and the tested measurements are of the same strength as the reference measurements; this is an obvious predicament. Therefore, it is justified to eliminate the four measurement cases from the attempts to identify two inaccurate measurements.

Where five or more measurements are made (be they any type), statistical detection of paired inaccurate measurements is possible but the method becomes quite cumbersome. Often graphical representation of the lines of positions and the corresponding landmarks would be more fruitful as performed in reference 28. The measurements are now grouped into $\binom{n}{2}$ different (n-2)-measurement subsets with the corresponding $\binom{n}{2}$ removed two-measurement groups. The (n-2)-measurement subsets can be used to calculate a posteriori reference variance estimate, s_R^2, with n-4 degrees of freedom. The removed two-measurement groups can be used to calculate two residuals which are each weighted by the corresponding element of the weighting matrix, $\underline{W}$. This allows the following ratio to be calculated $\binom{n}{2}$ times.

$$\frac{\displaystyle\sum_{i=n-2}^{n} \left[\frac{r_i^2}{\sigma_0^2}\right] w_{ii}}{\dfrac{(n-4)\, s_R^2}{\sigma_0^2}} = \frac{\chi_2^2}{\chi_{n-4}^2} \quad \text{for all combinations} \tag{E-4}$$

The numerator has two degrees of freedom, the denominator has n-4 degrees of freedom. The maximum of the $\binom{n}{1}$ ratios is tested against the F statistic at some level α. Thus

$$\left[\frac{\displaystyle\sum_{i=n-2}^{n} r_i^2 w_{ii}}{s_R^2}\right]_{max} > F_{2,n-4,\alpha} \tag{E-5}$$

The hypothesis (that the maximum ratio is not excessive) is rejected if the calculated ratio exceeds the critical $F_{2,n-4,\alpha}$ value. In any case where the hypothesis is rejected, the corresponding two-measurement subset, that was temporarily removed for testing, can be rejected. The remaining n-2 measurements can be further tested for compliance with any applicable standard, such as against the new χ^2_{n-2} value (n now being reduced by two from that in equation (E-1)). Simultaneous inaccuracy in more than one measurement can be partially avoided if no landmarks are used for more than one measurement.

Measurement combinations increase rapidly after the case $\binom{n}{2}$ so further statistical tests are superfluous. In fact $\binom{n}{2}$ leads to ten different measurement checks and might be superfluous itself. It is mentioned here only because the common landmark misplacement case can contribute significantly to position error (reference 2). The search for two outliers should be performed subsequent to the search for one outlier.

E.3 DIFFERENCE BETWEEN OBSERVED MEASUREMENTS AND COMPUTED MEASUREMENTS

In section 6.2.7, swd was discussed as a position error measure. Swd can also be used to detect large measurement errors. A test for outliers can be made on any subset of m measurements of the n measurement set. This test is as follows,

$$swd = \sum_{i=1}^{m} \frac{(\Delta m_i)^2}{\sigma_i^2} > \chi^2_{m,\alpha} \tag{E-6}$$

at some confidence level α for each m measurement subset. The number m would logically progress from one to two with the tests on many combinations $\binom{n}{1}$ and $\binom{n}{2}$, of measurements. Given that any test indicates an outlier, the outlier can be rejected, and the remaining measurements can be used to compute the position.

APPENDIX F

HP-41C AND OFFSHORE CALCULATOR-ASSISTED RESECTION

F.1 INTRODUCTION

Calculator routines were prepared to demonstrate analytical positioning using the HP-41C Programmable Calculator System. The system includes:

HP-41CV Programmable Calculator
HP-82153 Optical Wand
HP-82143A Thermal Printer

The report sections listed below were chosen for use in the demonstration:

3.3.1.3	4.2.1	5.1.1	6.2.3	8.2	C.1	D.1
3.3.2	4.2.2	5.2	6.2.4		C.2	D.2
	4.2.4				C.3.2	D.3
					C.4	D.4

Three sections follow in this appendix:

1. Section F.2 is a listing of the positioning program for the HP-41C System

2. Section F.3 is a brief flow chart to instruct the user how to operate the routine. The program and operating instructions are provided in a form usable by those already familiar with operation of the calculator.

3. Section F.4 is a bar-code generation routine developed for use on the HP9825A Calculator and the HP-9872B Plotter (adapted from Generating Barcode in the Hewlett-Packard Format, McNeal, Thomas, BYTE, January 1981). The routine can be used to generate optical bar-codes for the HP-41C programs and for the geodetic control data for landmarks and buoys. The routine requires the 23K-byte option on the HP-9825A and requires the following Read-Only-Memories to be inserted:

Advanced Programming	9872B Plotter
String Variables	General Input/Output
Extended Input/Output	

Once again, the routine has been provided in a form usable by those already familiar with the calculator and its operation.

The bar-code generation routine is useful in two different modes of operation, they are as follows:

Program Mode - In this mode, the routine is capable of creating complete pages of bar-code that represent calculator programs that can be loaded into memory by use of the wand. Once the data, programs, and special function keys have been loaded into the HP-9825A Calculator, the routine is operated by a list of commands that are assigned to the special function keys. The command and input format are given by File 14 of this appendix.

Geodetic Data Mode - In this mode, the calculator uses only subroutine DATA to generate the optical bar-code. The bar generated represents the landmark number, the latitude of the landmark, and the longitude of the landmark. The routine prompts the operator for the required input.

F.2 HP - 41C SUBROUTINE LISTINGS

```
PRP "P"

01♦LBL "P"
SF 07 CF 00 CF 04
FS? 06 GTO "M" CLRG
CF 29 FIX 0

10♦LBL 00
57.059 STO 12 "CP?"
XEQ 09

15♦LBL "M"
"LOP?" PROMPT STO 00
"TYP?" FS? 01 GTO 01
PROMPT STO 14 CLA
GTO 06

26♦LBL 01
2 STO 14

29♦LBL 06
"ENT " FIX 0 ARCL 00
GTO IND 14

34♦LBL 02
"├ L" 47.069 STO 12
XEQ 09 "ENT " FIX 0
ARCL 00 "├ P" 47.049
STO 12 XEQ 09 CLA
GTO 07

49♦LBL 03
"├ P" GTO 05

51♦LBL 04
"├ P"

53♦LBL 05
47.069 STO 12 XEQ 09
GTO 07

58♦LBL 09
FIX 7 FS? 00 GTO 11
AVIEW WADSCN 2.011
STO 13 0

67♦LBL 08
RCL IND 13 + ISG 13
GTO 08 256 MOD RCL 01
X≠Y? GTO 09 100
ST* 02 ST/ 05 ST/ 08
X↑2 ST/ 06 ST/ 10 100
* ST/ 07 ST/ 11
RCL 02 RCL 03 +
STO IND 12 FIX 0 PRX
FIX 7 ISG 12 RCL 04
RCL 05 + RCL 06 +
RCL 07 + PRX ADV
STO IND 12 ISG 12
RCL 08 RCL 09 +
RCL 10 + RCL 11 +
PRX ADV STO IND 12 RTN

110♦LBL 07
FIX 0 10 ST+ 14
GTO IND 14

127♦LBL 11
"├ ENTR" PRA PROMPT
X<>Y RCL Z STO IND 12
FIX 0 PRX FIX 7
XEQ 10 XEQ 10 RTN

136♦LBL 10
ISG 12 RDN PRX ADV
STO IND 12 RTN

143♦LBL 12
XEQ 22 RCL 47 1 E-7 *
+ XEQ 23 RCL 48
STO 02 RCL 49 STO 03
GTO 15

155♦LBL 13
156♦LBL 14
XEQ 22 XEQ 23 GTO 15

160♦LBL 22
RCL 47 1 E-3 * RCL 00
+ RTN

167♦LBL 23
RCL 00 20 + X<>Y
STO IND Y RCL 58
STO 06 RCL 59 STO 07
RCL 60 STO 04 RCL 69
STO 05 RTN

182♦LBL 15
6779206.4 STO 10
6.760658 E-3 STO 11
FIX 7 4 ST+ 14
XEQ IND 14 GTO "M"

192♦LBL 17
XEQ 32 .0175 * RCL 00
20 + RDN STO IND T
RDN RCL 00 50 + RDN
STO IND T 90 - XEQ 21
RCL 00 40 + RDN
STO IND T RCL 00 60 +
10 STO IND Y RTN

221♦LBL 18
XEQ 32 1.094 * RCL 00
50 + RDN STO IND T
RDN 100 + XEQ 21
RCL 00 40 + RDN
STO IND T RCL 00 60 +
.5 STO IND Y RCL 00
30 + 1 STO IND Y RTN

250♦LBL 16
XEQ 32 STO 10 X<>Y
STO 19 STO 01 RCL 04
X<> 02 STO 04 RCL 05
X<> 03 STO 05 XEQ 32
ST* 10 X<>Y ST- 19
ST+ 01 2.91 E-4 ST* 10
XEQ 25 XEQ 32 ST/ 10
X<>Y ST- 01 RCL 19 1
P-R R-P RDN X<0?
SF 05 ABS STO 19
RCL 01 XEQ 21 STO 01
90 FS?C 05 CHS -
XEQ 21 RCL 00 40 +
X<>Y STO IND Y RCL 00
50 + RCL 19 STO IND Y
RCL 00 30 + RCL 10
STO IND Y RCL 00 60 +
1 STO IND Y XEQ 25
RTN

313♦LBL 21
360 MOD RTN

317♦LBL 19
SIN X↑2 RCL 11 * CHS
1 + SQRT 1/X RCL 10
* RTN

330♦LBL 25
RCL 02 X<> 06 STO 02
RCL 03 X<> 07 STO 03
RTN

339♦LBL 32
RCL 07 RCL 05 -
STO 14 RCL 06 ENTER↑
XEQ 19 P-R STO 13 RDN
STO 15 RCL 04 ENTER↑
XEQ 19 P-R RCL 14
X<>Y P-R ST- 13 RDN
STO 14 RDN RCL 15 -
1 RCL 11 - * STO 15
RCL 06 COS * RCL 13
RCL 06 SIN * +
RCL 14 X<>Y R-P X<>Y
XEQ 21 RCL 13 RCL 15
R-P RCL 14 R-P R↑
X<>Y RTN END

PRP "O"

01♦LBL "O"
0 STO 98 ΣREG 13 CLΣ
XEQ "H" "HD " PROMPT
STO 58 FIX 0 ARCL X
PRA

13♦LBL 00
RCL 30 FS? 04 37
FC? 04 32 X=Y? GTO 01
XEQ "H" RCL IND 20
X=0? GTO 00 "MEAS "
RCL 30 10 MOD FIX 0
ARCL X PROMPT "├ "
FIX 2 ARCL X PRA
XEQ 03 RCL IND 50 -
RCL IND 60 1 X=Y?
GTO 10 RCL Z GTO 11

45♦LBL 10
RCL Z 60 *

49♦LBL 11
ENTER↑ X↑2 RCL IND 60
ST* Z X↑2 * ST+ 98
RDN RCL IND 30 X<>Y *
LASTX RCL IND 40 * Σ+
TONE 9 GTO 00

67♦LBL 03
RCL IND 60 .5 X=Y?
GTO 04 RCL Z ENTER↑
INT X<>Y FRC .6 / +
RTN

91♦LBL 04
RCL Z RTN

94♦LBL 01
RCL 15 RCL 01 *
RCL 00 RCL 04 * -
RCL 12 / STO 16
RCL 13 RCL 03 *
RCL 15 RCL 04 * -
RCL 12 / STO 14
FC? 04 GTO 05 RCL 98
RCL 13 RCL 14 * -
RCL 15 RCL 16 * -
RCL 05 2 - / ABS
SQRT STO 20

123♦LBL 05
RCL 58 RCL 60 +
RCL 40 P-R ST+ 16 RDN
ST+ 14 RCL 14 RCL 16
R-P FIX 0 1.09 * CLA
ARCL X "├ YDS " RDN
180 + XEQ 07 ARCL X
"├ T" "├ TO CP" PRA
CLA FC? 04 GTO "O"
GTO 06

153♦LBL 07
360 X<=Y? GTO 08 X<>Y
X<0? + RTN

161♦LBL 08
- RTN

164♦LBL 06
FIX 7 RCL 16 9.04 E-6
STO 09 * RCL 06 +
HMS "LA" ARCL X PRA
RCL 14 RCL 09 * CHS
RCL 06 COS / RCL 07
+ HMS "LO" ARCL X
PRA SF 05 FIX 2 10
STO 17 4.2 STO 18 3
STO 19 RCL 05 5 X<=Y?
STO 05 RCL 05 14 +
RCL IND X 1.09 *
ST* 20 XEQ 09 "A "
ARCL X "├ YDS" PRA
XEQ 09 "B " ARCL X
"├ YDS" PRA PI R-D
R↑ 2 / X<0? +
"DIR " ARCL X "├ T"
PRA GTO "O"

220♦LBL 09
RCL 01 RCL 03 +
RCL 04 CHS 2 *
RCL 01 RCL 03 - R-P
FS?C 05 CHS X<>Y RDN
+ 1/X 2 * ABS SQRT
RCL 29 * END

PRP "L"

01♦LBL "L"
CF 04 GTO "T"

04♦LBL "X"
SF 04

06♦LBL "T"
CF 06 FS? 10 GTO 03
12 STO 40 25 STO 60
1.007 STO 00

16♦LBL 00
RCL 00 40 + RCL IND X
ENTER↑ SIN X<>Y COS
10 ST- T RDN
RCL IND Z ST/ Y ST/ Z
RDN RCL Z 30 +
RCL IND X ST* Z ST* T
RDN 30 - X<>Y
STO IND Y RDN 10 +
X<>Y STO IND Y

48♦LBL 01
ISG 00 GTO 02 GTO 03

52♦LBL 02
RCL 00 20 + RCL IND Y
X≠0? GTO 01 GTO 00

60♦LBL 03
SF 10 ΣREG 00 CLΣ
XEQ "H"

65♦LBL 04
RCL 30 FS? 04 37
FC? 04 32 X=Y? GTO 05
XEQ "H" RCL IND 20
X≠0? GTO 04 TONE 9
RCL IND 30 RCL IND 40
Σ+ GTO 04

82♦LBL 05
RCL 03 RCL 01 *
RCL 04 X↑2 - STO 12
GTO "O" END
```

```
PRP "R"

 01*LBL "R"
SF 00  CF 07  CF 04
FS? 06  GTO "X"  CLRG
"CP? DATA"  PRA  PROMPT
X<>Y  RCL Z  FIX 0  PRX
FIX 7  RDN  PRX  HR
STO 06  RDN  PRX  HR
STO 07

 24*LBL "X"
"N.LOP"  PROMPT  ENTER^
INT  20  +  STO 00  RDN
STO IND 00  "TYPE?"
FS? 01  GTO 00  PROMPT
GTO 01

 39*LBL 00
2

 41*LBL 01
STO 01  40  RCL 00  +
GTO IND 01

 47*LBL 02
1  STO IND Y  GTO 05

 51*LBL 03
.5  STO IND Y  GTO 05

 55*LBL 04
10  STO IND Y

 58*LBL 05
RCL 00  30  +  "C?"
PROMPT  XEQ 06
STO IND Y  RDN  20  -
"G?"  PROMPT  STO IND Y
RDN  10  +  "GB?"
PROMPT  STO IND Y
GTO "V"

 79*LBL 06
ENTER^  INT  X<>Y  FRC
.6  /  +  END
```

```
PRP "S"

 01*LBL "S"
FS? 07  GTO "W"  FS? 00
GTO "X"  END
```

```
PRP "Q"

 01*LBL "Q"
CF 10  SF 12  ADV
"N.LOP"  PRA  "C<GAPG>"
PRA  21.027  STO 00

 11*LBL 00
CLA  SF 12  FIX 7
ARCL IND 00  ADV  PRA
CLA  CF 12  RCL 00  30
+  FIX 3  XEQ 03  RDN
"|-"  20  -  RCL IND X
ARCL X  RDN  "|-"  10  +
FIX 2  XEQ  CLA  ADV

 39*LBL 01
ISG 00  GTO 02  SF 06
FS? 07  GTO "W"  FS? 00
GTO "X"

 47*LBL 02
RCL IND 00  X≠0?  GTO 01
GTO 00

 52*LBL 03
RCL IND X  ENTER^  INT
X<>Y  FRC  .6  *  +
ARCL X  END
```

```
PRP "U"

 01*LBL "U"
FS? 01  GTO 01  SF 01
"C.S"  AVIEW  STOP

 08*LBL 01
CF 01  "ALL"  AVIEW  END
```

```
PRP "V"

 01*LBL "V"
FS? 03  GTO 01  SF 03
335  STO 60  "P"  AVIEW
STOP

 10*LBL 01
CF 03  25  STO 60  "S"
AVIEW  END
```

```
PRP "Y"

 01*LBL "Y"
FS? 00  GTO 01  SF 00
"W-ON"  AVIEW  STOP

 08*LBL 01
CF 00  "W-OFF"  AVIEW
END
```

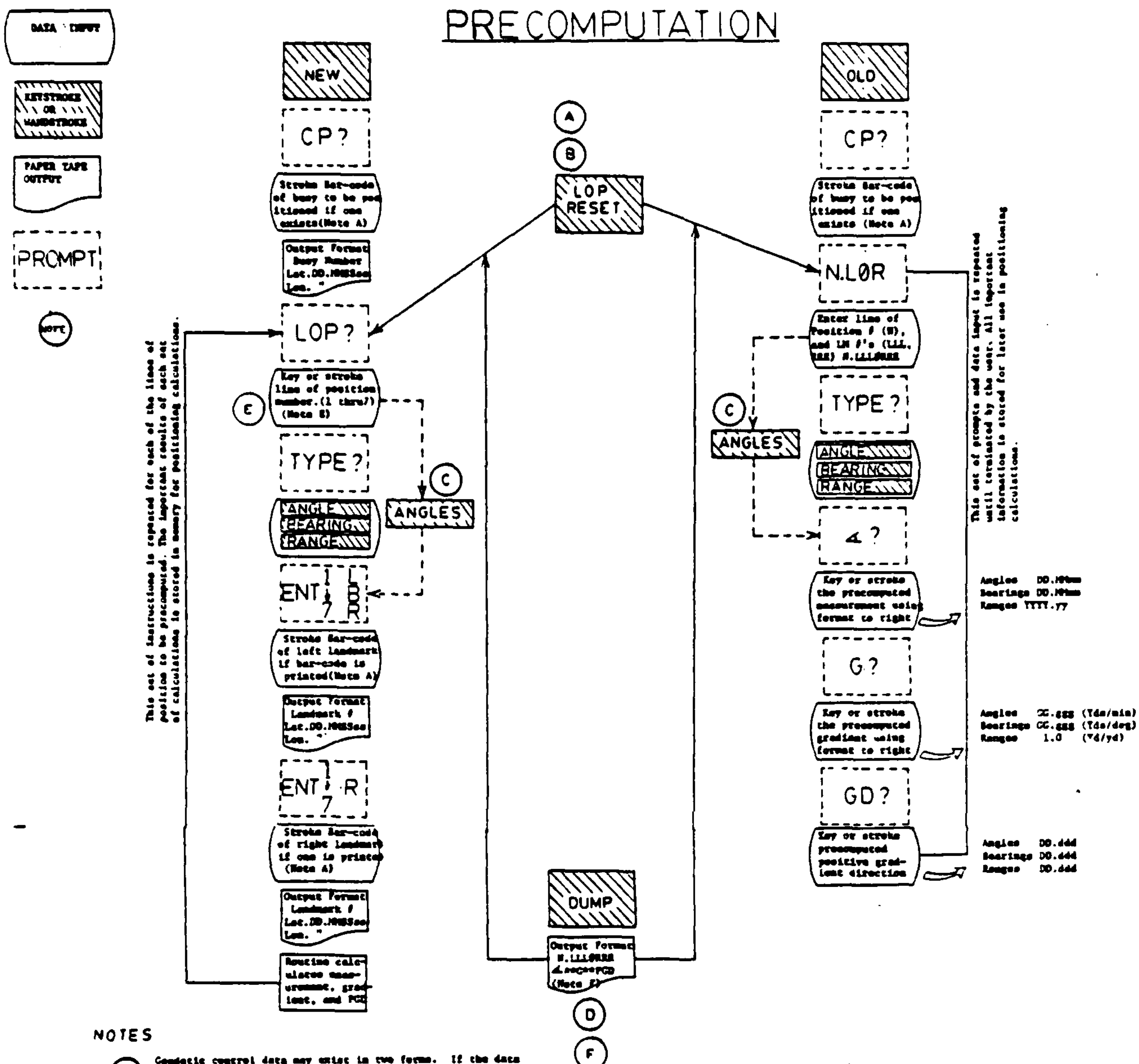
```
PRP "N"

 01*LBL "N"
20  STO 20  30  STO 30
40  STO 40  50  STO 50
60  STO 60  END
```

```
PRP "W"

 01*LBL "W"
1  ST+ 20  ST+ 30
ST+ 40  ST+ 50  ST+ 60
END
```

```
CAT 1
LBL'P
LBL'N
END        609 BYTES
LBL'Q
END        123 BYTES
LBL'L
LBL'K
LBL'T
END        163 BYTES
LBL'N
END        20 BYTES
LBL'W
END        19 BYTES
LBL'Q
END        430 BYTES
LBL'R
LBL'X
END        160 BYTES
LBL'Y
END        33 BYTES
LBL'U
END        20 BYTES
LBL'S
END        18 BYTES
LBL'V
END        31 BYTES
.END.      07 BYTES
```

```
PRKEYS

USER KEYS
 11 "P"
 12 "R"
 13 "S"
 14 "U"
 15 "Q"
 21 "T"
 22 "V"
 23 "L"
 24 "K"
 25 "Y"
 01 PRP
```

F.3 OPERATING INSTRUCTIONS FLOW CHARTS



NOTES

(A) Geodetic control data may exist in two forms. If the data exists in Bar-code form, follow the prompt by a stroke across the Bar-code of the desired buoy or landmark. If the data does not exist in bar code form, the WAND ON/OFF must be stroked or keyed. This allows data entry by keyboard or digit wand entry. Input data in the following format:

Key in buoy or landmark number 0 through 999
Press or stroke ENTER
Key in Latitude of buoy or landmark DD.MMSSss
Press or stroke ENTER
Key in Longitude of buoy or landmark DD.MMSSss Press or stroke CONTINUE

(B) If a data entry error is made when entering the charted position data, it is easiest to start over. If an error is made after the line of position number prompt occurs, then it is easiest to recover by pressing or stroking the LOP RESET and then re-enter the data for the line of position from that point on. You need not enter data for the other lines of position entered before the one in error, they have not been affected.

(C) If all measurements to be precomputed are angle measurements, the TYPE prompt can be suppressed by pressing or stroking ANGLES. Return to the normal mode by pressing the ANGLES key another time.

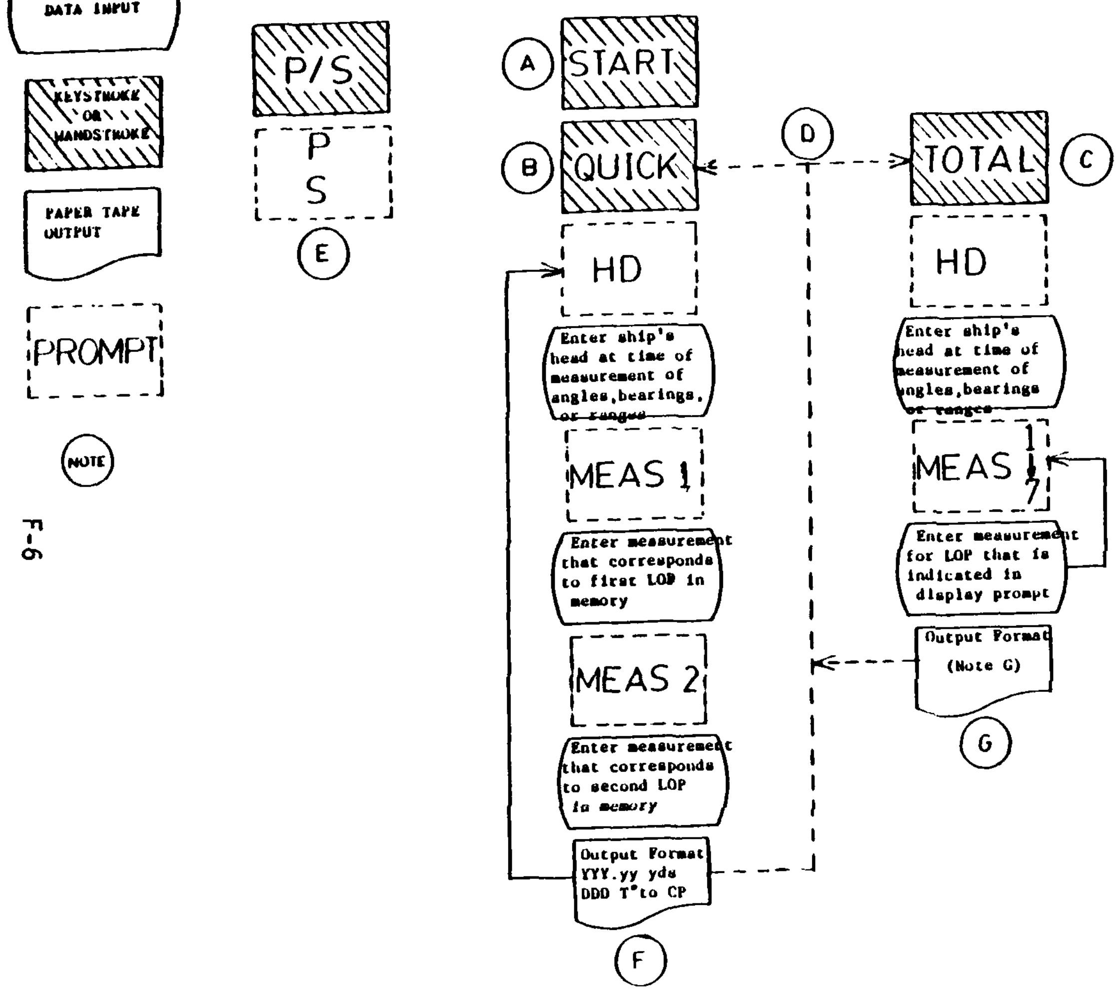
(D) New and old precomputed data can be merged after dumping the data entered first. Example: Enter data for two New lines of position using the New routine, then dump data. You can then press or stroke the OLD and enter previously computed data (or vice versa)

(E) The data entered for lines of position 1 and 2 are used for quick positioning calculations(conning guidance). The remaining line of position data sets are used only for total positioning calculations(positions determined using all lines of position in memory). At least three lines of position must be in memory before execution of a total position calculation or a computation error will occur.

(F) Data is dumped from memory in the same format that it is entered in the Old section above. Following a data dump, one can return to either the New or the Old routine for additional precomputations. Once positioning calculations have begun, further dumping of data using the dump routine is meaningless due to transform of memory contents into a more useful form for positioning calculations.

Drawn by M. A. Millbach on 10/21/81

POSITIONING



Drawn by M.A. Millbach on 10/21/81

NOTES

(A) Once START has been pressed or stroked, you may not return to the PRECOMPUTATION routine to add or subtract LOP data sets.

(B) A quick fix uses only the first two lines of position in memory and is performed automatically after the START activates the positioning routine.

(C) A total fix uses *all lines of position* in memory and will not be executed unless called by the user(this is done by stroking or pressing TOTAL)

(D) Quick and total fixes may be calculated in any order after the first quick fix by stroking or *pressing the respective* key.

(E) The P/S is stroked or keyed to initialize the routine with respect to the buoy port to be used in positioning. The starboard side is assumed unless P/S is stroked or keyed. Pressing or stroking the key again returns operation to the normal mode.

(F) The quick fix routine operates over and over again until the user is satisfied with the results, goes to a total fix, or terminates *calculator usage*. Each set of measurement inputs causes an output stating how far and in what direction the buoy port is from the charted position.

(G) The output of the total fix routine provides the following information in the format and units listed:

YYY.yy YDS DDD. T° TO CP
Latitude (La) DD.MMSSss
Longitude(Lo) DD.MMSSss
Major Semi-axis(A) YYY.yy YDS
Minor Semi-axis(B) YYY.yy YDS
Orientation of Major Semi-axis(DIR) DDD T°

F.4 HP - 9825/ HP - 9872 BAR-CODE GENERATION ROUTINE

FILE LIST	FUNCTION	NAME
File 0	INITIALIZATION ROUTINE	BAR-CODE
File 1	DATA REQUIRED TO RUN PROGRAM	DATA FILE
File 2	PROGAM BAR-CODE PLOTTING ROUTINE	PROG
File 3	SPECIAL FUNCTION KEY STORAGE	SPEC.FUNCT. KEYS
File 4	CONTROL OR PROMPTING ROUTINE	PROMPT
File 5	AUTOMATIC NUMBERING ROUTINE	NUMBER
File 6	PROGRAM LISTING ROUTINE	LIST
File 7	GEODETIC DATA BAR-CODE PLOTTING ROUTINE	DATA
File 8	COMPILATION ROUTINE	COMPILE
File 9	PROGRAM RENUMBERING ROUTINE	RENUMBER
File 10	empty file	
File 11	empty file	
File 12	PROGRAM STORAGE ROUTINE	SAVEPROG
File 13	PROGRAM RETRIEVAL ROUTINE	GETPROG
File 14	COMMAND DEFINITION ROUTINE	COMMANDS
File 15	BAR-CODE PLOTTING ROUTINE FOR USER DEFINED BIT PATTERNS	BARPLOT

SPECIAL FUNCTION KEY ASSIGNMENTS

f0	/YES	f8	/NUMBER	f16	/SECPROG
f1	*ldf4,0,0	f9	/COMPILE	f17	*ldp0,0
f2	/EXIT	f10	/DATA	f18	blank
f3	/SAVEPROG	f11	DELETE	f19	blank
f4	/PROG	f12	blank	f20	/RENUMBER
f5	/SCRATCH	f13	blank	f21	/ABORT
f6	/NO	f14	blank	f22	/BARPLOT
f7	/LIST	f15	/GETPROG		

FILE 0

```
0: "Bar-Code":
1: dim A$[1500],P$[750,2],M$[750,1],F$[750,2]
2: dim K[12],S[16],B[132],T$[50],U$[30],V$[30],S$[50],E$[132]
3: dim W$[27,1],I$[164,7],I[164],C$[60,1],C[60]
4: ldk 3
5: for I=1 to 750;fti(-1)→M$[I]→P$[I];next I
6: ""→A$
7: 0→T
8: trk 5;ctrk 8
9: ""→T$→U$→V$
10: ldf 1,W$,I$,I[*],C$,C[*]
11: ent "Command Definitions?",U$
12: if cap(U$)="N" or cap(U$)="NO";ldf 4,0
13: ldf 14,0
*27565
```

FILE 2

```
0: "Prog":
1: wrt 705,"VS25"
2: wrt 705,"IP100,100,9000,10000"
3: if flg8;jmp 2
4: prt "Compile Prog!";ldf 4,0
5: ent "Program Name(Limit 30 Letters)",T$
6: scl 0,6,0,10
7: csiz 1,2,1.25
8: 215→C;beep;wait 400;beep;wait 400;beep
9: fmt 8b,"Paper in Plotter",1ib
10: wrt 0,C,C,C,C,C,C,C,C,33,C,C,C,C,C,C,C,C,C;stp
11: pen# 1
12: plt 3.5,9.75,1
13: lbl T$
14: int(A/7)→X
15: if Amod7#0;X+1→X
16: plt 3,9.5,1
17: lbl "Number of Registers Needed  "
18: lbl X
19: 0→B→C→E→F→G→H→K→M→Z
20: 1→D
21: 1→L
22: inc B
23: "ITS":
24: itf(M$[C+1])→S[D+1]
25: C+1→C;D+1→D;E-1→E
26: if E#0;jmp 3
27: 0→F
28: gto "Bc"
29: if E<0;jmp 3
30: F+1→F
31: gto "Bc"
32: if S[D]#0;B+1→B
33: if itf(M$[C])>143;gto "2B"
34: "1B":
35: if itf(M$[C])#29 and itf(M$[C])#30;jmp 4
36: itf(M$[C+1])mod16+1→E
37: F+1→F
38: gto "Bc"
39: if itf(M$[C])<16 or itf(M$[C])>28;jmp 9
40: C+1→I
41: if itf(M$[I])<16 or itf(M$[I])>28;jmp 3
42: I+1→I
43: gto -2
44: I-C-1→E;F+1→F
45: if E#0;gto "Bc"
46: 0→F
47: gto "Bc"
48: 0→E→F
49: gto "Bc"
50: "2B":
51: if itf(M$[C])>207;gto "3B"
52: if itf(M$[C])<192 or itf(M$[C])>205;gto +7
53: F+1→F
54: if C+2<A;jmp 3
55: 2→E
56: gto "Bc"
57: itf(M$[C+2])mod16+2→E
58: gto "Bc"
59: 1→E
60: F+1→F
61: gto "Bc"
62: "3B":
63: if itf(M$[C])>240;jmp 4
64: 2→E
65: F+1→F
66: gto "Bc"
67: itf(M$[C])mod16→E
68: F+1→F
69: "Bc":
70: if D<16 and C<A;gto "ITS
```

FILE 2 (cont)

```
71: F→H;E→K
72: H mod16*16+Fmod16→S[3]
73: 16H+Gmod16→S[2]
74: for I=2 to D
75: M+S[I]mod256→M
76: if M>=256;Mmod256+1→M
77: next I
78: M→S[1]
79: "Convert":
80: G+1→G
81: for I=1 to D
82: S[I]→X
83: for Y=3+8I to 3+8(I-1) by -1
84: Xmod2→B[Y]
85: (X-B[Y])/2→X
86: next Y
87: next I
88: 0→B[1]→B[2]→B[8D+4]
89: 1→B[3D+3]
90: stp
91: 8D+4→J
92: plt 0,9.32-.5Gmod18,1
93: fxd 0
94: lbl "POW ";lbl G;lbl " ";lbl L;lbl " - ";lbl B;lbl " "
95: plt 0,9-.5Gmod18,1
96: for I=1 to J
97: if B[I]=0;iplt 0,.3,2;iplt 0,-.3,-1;jmp 2
98: iplt 0,.3,2;iplt 0,-.3,-1;iplt .018,0;iplt 0,.3,2;iplt 0,-.3,-1
99: iplt .04,0,1
100: next I
101: if Gmod18=0;dsp "Change Paper!!";stp
102: ina S,B
103: 3→D;B→L
104: if E=0;L+1→L
105: if C>A;gto "ITS"
106: prt "BarCode Complete"
107: ldf 4,0
*32110
```

FILE 4

```
0: "Prompt":
1: ent "",T$
2: pos(T$," ")→I
3: 0→V→K;cfg 1
4: 1→N
5: if I=0;gto "Command"
6: T$[1,I-1]→U$
7: if U$#"DELETE";jmp 4
8: T$[I+1,I+4]→U$
9: len(U$)+1→I
10: sfg 1
11: if I-1>4;prt "# too large";gto "Prompt"
12: for J=I-1 to 1 by -1
13: if U$[J,J]<"0" or U$[J,J]>"9";prt "?? # or Cmd";gto "Prompt"
14: V+(num(U$[J,J])-48)10↑K→V
15: K+1→K
16: next J
17: if V>750;prt "# too large";gto "Prompt"
18: if flg1#1;jmp 3
19: fti (-1)→P$[V]
20: gto "Prompt"
21: T$[I+1]→T$
22: A$&T$&"!"→A$
23: fti (T)→P$[V]
24: T+len(T$)+1→T
25: gto "Prompt"
26: "Command":
27: if T$="NO";gto "Prompt"
28: if T$="BARPLOT";ldf 15,0
29: if T$="DATA";ldf 7,0
30: if T$="NUMBER";ldf 5,0
31: if T$="RENUMBER";ldf 9,0
32: if T$="LIST";ldf 6,0
33: if T$="PROG";ldf 2,0
34: if T$="COMPILE";ldf 8,0
35: if T$="YES";ldf 14,0
36: if T$="SAVEPROG";ldf 12,0
37: if T$="GETPROG";ldf 13,0
38: if T$#"SCRATCH";jmp 5
39: ent "Do you really want to SCRATCH?",T$
40: if cap(T$)="NO" or cap(T$)="N";gto "Prompt"
41: for I=1 to 750;fti (-1)→M$[I]→P$[I];next I
42: 0→r6→r8→T;""→A$;gto "Prompt"
43: if T$="EXIT";dsp "Done";stp
44: if T$#"SECPROG";prt "??Unrec Cmd";gto "Prompt"
45: 2→N
46: ldf 2,0
*21262
```

FILE 5

```
0: "Number":
1: ent "Start",V
2: ent "Inc.",r9
3: if V>1000;ent "# too large";ldf 4,0
4: fxd 0
5: dsp "",V
6: enp "",T$
7: if T$="EDIT";ldf 4,0
8: A$&T$&""→A$
9: fti(T)→P$[V]
10: T+len(T$)+1→T
11: V+r9→V
12: jmp -8
+3419
```

FILE 6

```
0: "List":
1: fmt f3.0,c13
2: for I=1 to 750
3: if itf(P$[I])<0;gto +6
4: for J=1 to 50
5: if A$[itf(P$[I])+J,itf(P$[I])+J]="";jmp 2
6: next J
7: A$[itf(P$[I])+1,itf(P$[I])+J-1]→S$
8: wrt 16,I,S$
9: next I
10: ldf 4,0
+12758
```

FILE 7

```
0: "Data":inc 5,K;0→r10→r11→r12→r13;""→T$
1: wrt 705,"VS25"
2: 200→A
3: enp "LM # ?",r10
4: enp "LM Name?(50 C's)",T$
5: enp "Latitude?(DD.MMSSsss)",r11;prnd(r11,-6)→r11
6: enp "Longitude?(DD.MMSSsss)",r12;prnd(r12,-6)→r12
7: ent "Location of Bar-Code?",r14;if r14>16;jmp 0
8: jmp r14
9: wrt 705,"IP 000,7000,7900,8000";gto "Plot"
10: wrt 705,"IP000,6000,7900,7000";gto "Plot"
11: wrt 705,"IP000,5000,7900,6000";gto "Plot"
12: wrt 705,"IP000,4000,7900,5000";gto "Plot"
13: wrt 705,"IP000,3000,7900,4000";gto "Plot"
14: wrt 705,"IP000,2000,7900,3000";gto "Plot"
15: wrt 705,"IP000,1000,7900,2000";gto "Plot"
16: wrt 705,"IP000,000,7900,1000";gto "Plot"
17: wrt 705,"IP5100,7000,13000,8000";gto "Plot"
18: wrt 705,"IP5100,6000,13000,7000";gto "Plot"
19: wrt 705,"IP5100,5000,13000,6000";gto "Plot"
20: wrt 705,"IP5100,4000,13000,5000";gto "Plot"
21: wrt 705,"IP5100,3000,13000,4000";gto "Plot"
22: wrt 705,"IP5100,2000,13000,3000";gto "Plot"
23: wrt 705,"IP5100,1000,13000,2000";gto "Plot"
24: wrt 705,"IP5100,000,13000,1000";gto "Plot"
25: "Plot":
+9155
```

FILE 7 (cont)

```
26: pen# 1
27: scl 0,7,0,1
28: csiz 10,2,.125
29: "Label":
30: 215→C;beep;wait 400;beep;wait 400;beep
31: fmt 8b,"Paper in Plotter",11b
32: wrt 0,C,C,C,C,C,C,C,C,33,C,C,C,C,C,C,C,C,C,C;stp
33: plt .3,.7,1;lbl "LM ";fxd 0;lbl r10;lbl " ";lbl T$
34: plt .3,.55,1;lbl "Lat ";fxd 7;lbl r11
35: plt 2,.55,1;lbl "Lon ";lbl r12
36: "Number":
37: int(r10/100)→K[2]
38: 100frc(r10/100)→K[3]
39: "LatLon":
40: for I=11 to 12
41: int(rI)→K[4+4(I-11)]
42: int(100frc(rI))→K[5+4(I-11)]
43: int(100frc(100rI))→K[6+4(I-11)]
44: int(100frc(10000rI))→K[7+4(I-11)]
45: next I
46: "Chksum":
47: 0→r13
48: for I=2 to 11
49: r13+K[I]→r13
50: next I
51: r13mod256→K[1]
52: "Concert":
53: for I=1 to 11
54: K[I]→X
55: for Y=2+8I to 3+8(I-1) by -1
56: Xmod2→B[Y]
57: (X-B[Y])/2→X
58: next Y
59: next I
60: 0→B[1]→B[2]→B[92]
61: 1→B[91]
62: "Plt":
63: plt .3,.2,1
64: for I=1 to 92
65: if B[I]=0;iplt 0,.3,2;iplt 0,-.3,-1;jmp 3
66: iplt 0,.3,2;iplt 0,-.3;iplt .015,0;iplt 0,.3;iplt 0,-.3,-1
67: iplt .03,0,1
68: next I
69: plt 0,0,1;iplt 4.5,0,2;iplt 0,1;iplt -4.5,0;iplt 0,-1
70: pen# 0
71: enp "More LM's?",T$
72: if cap(T$)="N" or cap(T$)="NO";ldf 4,0
73: gto "Data"
+4670
```

FILE 8

```
0: "Compile":1→S
1: ""→S$
2: for J=1 to 750
3: if itf(P$[J])<0;gto "next J"
4: for R=1 to 50
5: if A$[itf(P$[J])+R,itf(P$[J])+R]="";jmp 2
6: next R
7: A$[itf(P$[J])+1,itf(P$[J])+R-1]→T$
8: S→r0
9: "Scan":
10: if T$="ABORT";ldf 4,0
11: ""→U$→V$
12: 0→r1→r2
13: -1→V
14: if flg6#1;jmp 3
15: fti(0)→M$[S]
16: S+1→S
17: cfg 1;cfg 2;cfg 3;cfg 4;cfg 5;cfg 6
18: T$→S$
19: if T$="END" or T$=".END.";gto "End"
20: if T$[1,1]#"'" and T$[1,2]#"A'";jmp 14
21: if T$[1,2]#"A'";jmp 3
22: T$[2]→T$;prt T$
23: sfg 1
24: len(T$)→L
25: if L>15;jmp 3
```

FILE 8 (cont)

```
26: prt "Line too long! #",J
27: jmp 3
28: if T$[L,L]="'";jmp 4
29: prt "Alpha Error #",J
30: gsb "Error"
31: gto "Scan"
32: sfg 2
33: gto "IS"
34: for I=1 to len(T$)
35: T$[I,I]→U$
36: if U$>="0" and U$<="9";jmp 4
37: if (U$="+" or U$="-") and len(T$)>1;jmp 3
38: if U$=" " or U$="E" or U$=".";jmp 2
39: jmp 4
40: next I
41: sfg 6
42: gto "IS"
43: pos(T$," ")→r1
44: if r1#0;jmp 3
45: sfg 3
46: gto "IS"
47: T$[r1+1]→U$
48: T$[1,r1-1]→T$
49: len(U$)→L
50: if U$[1,1]#"'" or U$[L,L]#"'";jmp 5
51: if L>9;gto -25
52: U$[2,L-1]→U$
53: sfg 4
54: gto "IS"
55: pos(U$," ")→r2
56: if r2=0;jmp 8
57: U$[1,r2-1]→V$
58: if V$="IND";jmp 4
59: prt "Operand Error #",J
60: gsb "Error"
61: gto "Scan"
62: sfg 5
63: U$[r2+1]→U$
64: if len(U$)<=2;gto "IS"
65: prt "Num. Op. Error #",J
66: gsb "Error"
67: gto "Scan"
68: "IS":
69: if flg2#1;jmp 26
70: fti (240+L-2)→M$[S]
71: if flg1=1;fti (itf(M$[S])+1)→M$[S]
72: S+1→S
73: 1→X
74: 59→Y
75: if flg1#1;jmp 3
76: fti (127)→M$[S]
77: S+1→S
78: for I=2 to L-1
79: X→B;Y→C
80: int((B+C)/2)→M
81: if T$[I,I]=C$[M];jmp 5
82: if T$[I,I]>C$[M];M+1→B
83: if T$[I,I]<C$[M];M-1→C
84: if B<=C;jmp -4
85: 0→M
86: M→Z
87: if Z#0;jmp 4
88: prt "Char. Error #",J
89: gsb "Error"
90: gto "Scan"
91: fti (C[Z])→M$[S]
92: S+1→S
93: next I
94: gto "next J"
95: if flg6#1;gto "Other"
96: "Dig":cfg 9;if T$="ABORT";ldf 4,0
97: if T$[1,1]#"+" and T$[1,1]#"-";jmp 5
98: if T$[1,1]#"-";jmp 3
99: fti (28)→M$[S]
100: S+1→S
101: T$[2]→T$
102: pos(T$," ")→r3
103: pos(T$,"E")→r4
104: if r3=0;len(T$)→r3
105: if r4#0;r4-1→r3
106: for I=1 to r3
107: if T$[I,I]#".";jmp 6
108: if flg9;jmp 11
109: sfg 9
110: fti (26)→M$[S]
111: S+1→S
112: jmp 4
113: if T$[I,I]<"0" or T$[I,I]>"9";jmp 3
114: fti (num(T$[I,I])-32)→M$[S]
115: S+1→S
116: next I
117: if r3=len(T$) and r4=0;gto "next J"
118: if I=r3 or I=r4;jmp 4
119: prt "Dig. Error #",J
120: gsb "Error"
```

FILE 8 (cont)

```
121: gto "Dig"
122: T$[I]→T$
123: if T$[1,1]#"E";jmp -4
124: fti (27)→M$[S]
125: S+1→S
126: T$[2]→T$
127: if T$[1,1]#"-";jmp 3
128: fti (28)→M$[S]
129: S+1→S
130: if T$[1,1]="-" or T$[1,1]="+";T$[2]→T$
131: T$[I]→T$
132: len(T$)→L
133: if L>2;jmp -14
134: for I=1 to L
135: if T$[I,I]<"0" or T$[I,I]>"9";jmp -16
136: fti (num(T$[I,I])-32)→M$[S]
137: S+1→S
138: next I
139: gto "next J"
140: "Other":
141: len(U$)→L
142: if flg3;gto "All Other"
143: if flg4;gto "Special"
144: if L<=2;jmp 4
145: prt "Num. Too Long #",J
146: gsb "Error"
147: gto "Scan"
148: if L<2;jmp 6
149: (num(U$[1,1])-48)10+num(U$[2,2])-48→V
150: if V>=0 and V<=99;gto "Special"
151: prt "Oper. Error #",J
152: gsb "Error"
153: gto "Scan"
154: 0→V
155: for I=1 to 26
156: if U$=W$[I];I+101→V
157: next I
158: if V#0;gto "Special"
159: num(U$)-48→V
160: if V>=0 and V<=9;gto "Special"
161: prt "Stk/Dig Op Err #",J
162: gsb "Error"
163: gto "Scan"
164: "Special":
165: if flg5;128+V→V
166: if T$#"GTO" and T$#"XEQ";gto "Sto"
167: if flg5#1;jmp 6
168: fti (174)→M$[S]
169: if T$="GTO";V-128→V
170: fti (V)→M$[S+1]
171: S+2→S
172: gto "next J"
173: if flg4#1;gto "SF Gto"
174: 1→X
175: 59→Y
176: fti (29)→M$[S]
177: if T$="XEQ";fti (30)→M$[S]
178: len(U$)→L
179: fti (240+L)→M$[S+1]
180: "Char":
181: for I=1 to L
182: X→B;Y→C
183: int((B+C)/2)→M
184: if U$[I,I]=C$[M];jmp 5
185: if U$[I,I]>C$[M];M+1→B
186: if U$[I,I]<C$[M];M-1→C
187: if B<=C;jmp -4
188: 0→M
189: M→Z
190: if Z#0;jmp 4
191: prt "Bad Alpha Lbl #",J
192: gsb "Error"
193: gto "Scan"
194: fti (C[Z])→M$[S+1+I]
195: next I
196: S+L+2→S
197: gto "next J"
198: "SF Gto":
199: if V>14 or T$="XEQ";jmp 5
200: fti (177+V)→M$[S]
201: fti (0)→M$[S+1]
202: S+2→S
203: gto "next J"
204: fti (208)→M$[S]
205: if T$="XEQ";fti (224)→M$[S]
206: fti (0)→M$[S+1]
207: fti (V)→M$[S+2]
208: S+3→S
209: gto "next J"
210: "Sto":
211: if T$#"STO";jmp 9
212: if V>15;jmp 4
213: fti (V+48)→M$[S]
214: S+1→S
215: gto "next J"
```

FILE 8 (cont)

```
216: fti (145)→M$[S]
217: fti (V)→M$[S+1]
218: S+2→S
219: gto "next J"
220: if T$#"RCL";jmp 9
221: if V>15;jmp 4
222: fti (32+V)→M$[S]
223: S+1→S
224: gto "next J"
225: fti (144)→M$[S]
226: fti (V)→M$[S+1]
227: 2+S→S
228: gto "next J"
229: if T$#"LBL";gto "All Other"
230: if flg4#1;jmp 10
231: 1→X
232: 59→Y
233: fti (192)→M$[S]
234: fti (0)→M$[S+1]
235: len(U$)→L
236: fti (L+241)→M$[S+2]
237: fti (0)→M$[S+3]
238: S+2→S
239: gto "Char"
240: if V>14;jmp 4
241: fti (1+V)→M$[S]
242: 1+S→S
243: gto "next J"
244: fti (207)→M$[S]
245: fti (V)→M$[S+1]
246: S+2→S
247: gto "next J"
248: "All Other":
249: 1→X
250: 164→Y
251: X→B;Y→C
252: int((B+C)/2)→M
253: if T$=I$[M];jmp 5
254: if T$>I$[M];M+1→B
255: if T$<I$[M];M-1→C
256: if B<=C;jmp -4
257: 0→M
258: M→Z
259: if Z#0;jmp 4
260: prt "Bad Inst. #",J
261: gsb "Error"
262: gto "Scan"
263: if frc(I[Z])=0;jmp 4
264: fti (int(I[Z]))→M$[S]
265: fti (1000frc(I[Z]))→M$[S+1]
266: S+2→S;jmp 3
267: fti (I[Z])→M$[S]
268: S+1→S
269: if I[Z]<64 or I[Z]>143 or flg3;jmp 4
270: prt "Extraneous Op #",J
271: gsb "Error"
272: gto "Scan"
273: if I[Z]<144 or I[Z]>159 and I[Z]<168;gto "next J"
274: if I[Z]<144 or I[Z]>191 or V#0;jmp 4
275: prt "Missing Operand #",J
276: gsb "Error"
277: gto "Scan"
278: fti (V)→M$[S]
279: S+1→S
280: "next J":
281: next J
282: "End":
283: fti (192)→M$[S]
284: fti (0)→M$[S+1]
285: fti (47)→M$[S+2]
286: S+2→A
287: prt "Compiled"
288: sfg 8
289: ldf 4,0
290: "Error":
291: r0→S
292: prt "Inst. given was",S$
293: prt "Enter correct"
294: prt "Inst.(No Line#)"
295: prt "To abort,type"
296: ent "ABORT?",T$
297: if T$="ABORT";ret
298: A$&T$&";"→A$
299: fti (T)→P$[J]
300: T+len(T$)+1→T
301: ret
302: end
*15166
```

INSTRUCTION MNEMONICS AND NUMERIC VALUES

FILE 1

1	%	76.000
2	%CH	77.000
3	Σ+	71.000
4	Σ-	72.000
5	ΣREG	153.000
6	*	66.000
7	+	64.000
8	-	65.000
9	/	67.000
10	1/X	96.000
11	10↑X	87.000
12	7CLREG	167.203
13	7DSP0	167.204
14	7DSP1	167.205
15	7DSP2	167.206
16	7DSP3	167.207
17	7DSP4	167.208
18	7DSP5	167.209
19	7DSP6	167.210
20	7DSP7	167.211
21	7DSP8	167.212
22	7DSP9	167.213
23	7DSPI	167.214
24	7DSZ	167.215
25	7DSZI	167.216
26	7ENG	167.217
27	7FIX	167.218
28	7GSBI	167.219
29	7GTOI	167.220
30	7ISZ	167.221
31	7ISZI	167.222
32	7P[illegible]S	167.223
33	7PPREG	167.224
34	7PRSTK	167.225
35	7PRX	167.226
36	7PCL&	167.227
37	7SCI	167.228
38	ABS	97.000
39	ACA	167.065
40	ACCHR	167.066
41	ACCOL	167.067
42	ACOS	93.000
43	ACSPEC	167.068
44	ACX	167.069
45	ADV	143.000
46	AOFF	139.000
47	AON	140.000
48	ARCL	155.000
49	ASHF	136.000
50	ASIN	92.000
51	ASTO	154.000
52	ATAN	94.000
53	AVIEW	126.000
54	BEEP	134.000
55	BLDSPEC	167.070
56	CF	169.000
57	CHS	84.000
58	CL&	112.000
59	CLA	135.000
60	CLD	127.000

61	CLRG	138.000
62	CLST	115.000
63	CLX	119.000
64	COS	90.000
65	D-R	106.000
66	DEC	95.000
67	DEG	128.000
68	DSE	151.000
69	ENG	158.000
70	ENTER↑	131.000
71	E↑X	85.000
72	E↑X-1	88.000
73	FACT	98.000
74	FC?	173.000
75	FC?C	171.000
76	FIX	156.000
77	FRC	105.000
78	FS?	172.000
79	FS?C	170.000
80	GRAD	130.000
81	HMS	108.000
82	HMS+	73.000
83	HMS-	74.000
84	HR	109.000
85	INT	104.000
86	ISG	150.000
87	LASTX	118.000
88	LN	80.000
89	LN1+X	101.000
90	LOG	86.000
91	MEAN	124.000
92	MOD	75.000
93	MRG	167.193
94	OCT	111.000
95	OFF	141.000
96	P-R	78.000
97	PI	114.000
98	PP&	167.082
99	PRA	167.072
100	PRAXIS	167.073
101	PRBUF	167.074
102	PRFLAGS	167.075
103	PRKEYS	167.076
104	PROMPT	142.000
105	PRPLOT	167.078
106	PRPLOTP	167.079
107	PRREG	167.080
108	PRREGX	167.081
109	PRSTK	167.083
110	PRX	167.084
111	PSE	137.000
112	R-D	107.000
113	R-P	79.000
114	RAD	129.000
115	RCL	144.000
116	RDN	117.000
117	RDTA	167.194
118	RDTAX	167.195
119	REGPLOT	167.085
120	RND	110.000
121	RSUB	167.196
122	RTN	133.000
123	R↑	116.000
124	SCI	157.000
125	SDEV	125.000

126	SF	168.000
127	SIGN	122.000
128	SIN	89.000
129	SKPCHR	167.086
130	SKPCOL	167.087
131	SQRT	82.000
132	ST*	148.000
133	ST+	146.000
134	ST-	147.000
135	ST/	149.000
136	STKPLOT	167.088
137	STO	145.000
138	STOP	132.000
139	TAN	91.000
140	TONE	159.000
141	VIEW	152.000
142	WDTA	167.199
143	WDTAX	167.200
144	WNDDTA	166.193
145	WNDDTX	166.194
146	WNDLNK	166.195
147	WNDSCN	166.197
148	WNDSUB	166.196
149	WNDTST	166.198
150	WSTS	167.202
151	X#0?	99.000
152	X#Y?	121.000
153	X<0?	102.000
154	X<=0?	123.000
155	X<=Y?	70.000
156	X<>	206.000
157	X<>Y	113.000
158	X<Y?	68.000
159	X=0?	103.000
160	X=Y?	120.000
161	X>0?	100.000
162	X>Y?	69.000
163	X↑2	81.000
164	Y↑X	83.000

LOCAL LABEL AND STACK REGISTER CHARACTERS

1	A	21	
2	B	22	a
3	C	23	b
4	D	24	c
5	E	25	d
6	F	26	e
7	G	27	
8	H		
9	I		
10	J		
11	T		
12	Z		
13	Y		
14	X		
15	L		
16			
17			
18			
19			
20			

VALID HP-41C CHARACTERS AND CHARACTER CODE

1		32.000
2	#	35.000
3	$	36.000
4	%	37.000
5	[illegible]	38.000
6	*	42.000
7	+	43.000
8	,	44.000
9	-	45.000
10	.	46.000
11	/	47.000
12	0	48.000
13	1	49.000
14	2	50.000
15	3	51.000
16	4	52.000
17	5	53.000
18	6	54.000
19	7	55.000
20	8	56.000
21	9	57.000
22	:	58.000
23	;	59.000
24	<	60.000
25	=	61.000
26	>	62.000
27	?	63.000
28	@	64.000
29	A	65.000
30	B	66.000
31	C	67.000
32	D	68.000
33	E	69.000
34	F	70.000
35	G	71.000
36	H	72.000
37	I	73.000
38	J	74.000
39	K	75.000
40	L	76.000
41	M	77.000
42	N	78.000
43	O	79.000
44	P	80.000
45	Q	81.000
46	R	82.000
47	S	83.000
48	T	84.000
49	U	85.000
50	V	86.000
51	W	87.000
52	X	88.000
53	Y	89.000
54	Z	90.000
55	↑	94.000
56	a	97.000
57	b	98.000
58	c	99.000
59	d	100.000
60	e	101.000

FILE 9

```
0: "Renumber":
1: ent "Old Start#",O
2: ent "NewStart",P
3: ent "Increment",Q
4: for I=P+1 to O-1
5: if itf(P$[I])=-1;jmp 2
6: prt "Error-Overwrite Inst.";ldf 4,0
7: next I
8: for I=1 to 750
9: P$[I]→K$[I]
10: if I>=O;fti (-1)→P$[I]
11: next I
12: O-1→K
13: for I=P to 750 by Q
14: K+1→K
15: if K>750;ldf 4,0
16: if itf(K$[K])<0;jmp -2
17: K$[K]→P$[I]
18: next I
19: prt "Error-# too large"
20: for I=1 to 750
21: K$[I]→P$[I]
22: next I
23: ldf 4,0
+25682
```

FILE 12

```
0: "Saveprog":
1: enp "Save on which file#(>15)?",r10
2: rcf r10,A$,P$,M$
3: ldf 4,0
+14414
```

FILE 13

```
0: "Getprog":
1: enp "Get from which file#?(>15)",r10
2: sfg 8
3: ldf r10,A$,P$,M$
4: ldf 4,0
+23007
```

FILE 15

```
0: "Barplot":cfg 0
1: enp "Bar Title",T$
2: enp "Number of Bytes?",B
3: ent "Decimal(10) or Binary(2)",D
4: "Pos":
5: enp "Vert.Pos.#(1-20)",R
6: enp "Hor.Pos.#(1-15)",H
7: if flg0;gto "Plot"
8: if D=10;gto "Decimal"
9: "Binary":
10: for I=1 to 8B
11: ent B[I]
12: next I
13: for I=8B to 1 by -1
14: B[I]→B[I+2]
15: next I
16: 0→B[1]→B[2]→B[8B+4]
17: 1→B[8B+3]
18: gto "Plot"
19: "Decimal":
20: for I=1 to B
21: prt "Byte",I
22: enp M;fti (M)→M$[I]
23: next I
24: for I=1 to B
25: itf(M$[I])→X
26: for Y=2+8I to 3+8(I-1) by -1
27: Xmod2→B[Y]
28: (X-B[Y])/2→X
29: next Y
30: next I
31: 0→B[1]→B[2]→B[8B+4]
32: 1→B[8B+3]
33: "Plot":
34: wrt 705,"IP 000,000,11000,8000"
35: scl 0,11,0,8
36: csiz 1.5,2,8/11
37: if 2(16-H)>B;jmp 2
38: prt "No Room!!";sfg 0;gto "Pos"
39: 216→C;beep;wait 400;beep;wait 400;beep
40: fmt 3b,"Paper in Plotter",1b
41: wrt 0,C,C,C,C,C,C,C,C,33,C,C,C,C,C,C,C,C,C,C;str
42: pen# 1
43: plt .5(H-1),.5(R-1)
44: wrt 705,"PD"
45: pen
46: enp "Position Good?(Yes or No)",U$
47: if cap(U$)="NO" or cap(U$)="N";sfg 0;gto "Pos
48: iplt 0,.35,1;lbl T$
49: plt .5(H-1),.5(R-1)
50: for I=1 to 8B+4
51: if B[I]=0;iplt 0,.3,2;iplt 0,-.3,-1;jmp 2
52: iplt 0,.3,2;iplt 0,-.3,-1;iplt .015,0;iplt 0,.3,2;iplt 0,-.3,-1
53: iplt .04,0,1
54: next I
55: enp "Bar Complete-Any more?",U$
56: if cap(U$)="YES" or cap(U$)="Y";gto "Barplot"
57: ldf 4,0
+21517
```

FILE 14

```
0: "Commands":
1: wrt 6,"Commands available in this program are:"
2: wrt 6,""
3: wrt 6,"   ABORT-Error routine exit command"
4: wrt 6,"   BARPLOT-Generates a row of user defined bar code"
5: wrt 6,"   at a user defined location on plotting surface"
6: wrt 6,"   COMPILE-Compiles the 41C Progam currently entered"
7: wrt 6,"   DATA-Plots barcode representing LM#,Lat,and Lon"
8: wrt 6,"   DELETE nn -Deletes the numbered instruction from the program"
9: wrt 6,"   EXIT- Halts this program or stops the number generator"
10: wrt 6,"   GETPROG- Retrieves the compiled code from cassette tape"
11: wrt 6,"   LIST- Lists the entire 41C program currently in memory"
12: wrt 6,"   NUMBER- Generates 41C instruction numbers-stopped by EXIT"
13: wrt 6,"   RENUMBER- Renumbers the 41C program instruction numbers"
14: wrt 6,"   PROG- Generates the barcode for HP-41C Program"
15: wrt 6,"   PROMPT-Loads prompt subroutine if calculator is stopped"
16: wrt 6,"   RESET-Clears program to start over"
17: wrt 6,"   SECPROG- Generates the bar-code for a private program"
18: wrt 6,"   SAVEPROG- Stores the compiled code on cassette tape"
19: wrt 6,"   SCRATCH- Erases the entire 41C program"
20: wrt 6,"   YES/NO- YES lists the commands available"
21: wrt 6,""
22: wrt 6,"Syntax for instruction entry:"
23: wrt 6,"  A)Instruction format"
24: wrt 6,"      n (41C instruction)"
25: wrt 6,"      (Blanks are delimiters)"
26: wrt 6,"  B)Special Symbols:"
27: wrt 6,"      1) Use '&' instead of sigma sign"
28: wrt 6,"      2) Use '@' instead of angle sign"
29: wrt 6,"      3) Use '#' instead of the not equal sign"
30: wrt 6,"      4) Use single vice double quotes"
31: wrt 6,"  C)Text Format:   ' Text Entry>'  (Note single quotes)"
32: wrt 6,"            or    A ' Text Entry>'  (For appending text)"
33: ldf 4,0
+498
```